Information processing and mathematics learning disabilities[1]

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Abstract *: Successful mathematics learning requires the efficient processing of the information that defines the arithmetic tasks. Information processing relates to the ways in which individuals make sense of, or interpret, the information to which they are exposed.*

The present study examines four aspects of information processing and their relationship for whole number computation for third and fifth grade students. The aspects included students' ability to (1) manipulate numerals, (2) encode number sentences , (3) recognise order among numbers and (4) perform an arithmetic procedure.

Information processing in each area correlated with computational skill. At risk students were less efficient in their information processing. As well, the complexity of the numerical information affected how well the students could use it. The more complex the numerical information was, the greater the load it placed on the learner. The implications for diagnosing low mathematics achievement are discussed.

The ability to complete arithmetic computational skills such as 74 + **r** = 92 and 3.4 x .8 =**u** requires a range of related prerequisite knowledge. The completion of 74 + **r** = 92 requires students to recall and / or apply single and 2- digit addition and subtraction procedures, while 3.4 x .8 =**u** requires the recall of multiplication facts and decimal place value concepts. Most approaches to the remediation of arithmetic disabilities recognise the need for this prerequisite arithmetic knowledge to be 'in place'. Intervention teaching and pedagogy usually recognises this and teaches or at least stimulates the necessary prerequisite arithmetic knowledge.

A second area of prerequisite mathematics knowledge that is frequently overlooked in understanding the nature of particular arithmetic disabilities is the efficient processing of the information that defines the arithmetic task. Information processing relates to the ways in which individuals make sense of, or interpret, the information to which they are exposed (Schweizer, 1998). A number of factors influence the efficiency with which

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they do this; their familiarity with the information, its comparative complexity, the number of processing steps it requires, the effort they need to invest in processing it (Crawford, 1991), extent to which the information needs to be processed and retained simultaneously, the demand it makes on working memory and the amount of stimulus information (Schweizer, 1993).

In order to complete the tasks 74 + **r** = 92 and 3.4 x .8 =**u**, learners need to read (that is, encode) each symbol and each statement, recognise order among the numbers, categorise each task in terms of what they already know, use 'mental actions' that match the arithmetic operations and recall number facts (Kaufmann, 2002; McNeil & Burgess, 2002; Temple, 1992).

These investigators propose that the various aspects operate mutually independently and provide outputs to a synthesis or integration mechanism. Some students, for example, can apply algorithms correctly only when they are provided with the relevant written number facts. Some have accurate number processing skills but don't have other components of the model in place (McNeil & Burgess, 2002) . Some may recall number facts accurately but have difficulty calculating (Kaufmann, 2002) while others can apply procedures but not recall the number facts (Temple, 1991). Some can comprehend and produce numbers but have difficulty recalling numerical facts or doing simple arithmetic computations (Shalev, Weirtman & Amir, 1988). This study examines the influence of these information processing skills on mathematics task completion.

One aspect of arithmetic information processing is being able to read written numerals. Students read numerical symbols in different ways; they can (1) say aloud or 'transcode' the written numerals; (2) recognise a written numeral they have seen earlier; or (3) match a numeral with a quantity. The present study uses the second type of task. Students are shown a target numeral and a set of numerals and need to select the one seen earlier. The ease with which they can make this selection is assumed to be determined by their ability to form an impression of the numeral seen earlier. Suppose, for example, the target numeral comprises 3 digits. Students can use different procedures to select the second instance of the numeral. These range from encoding the target number by noting and retaining the three places at once to matching separate digits one at a time. Encoding the number directly is assumed to be more effective than discriminating by matching each digit at a time.

A second aspect relates to being able to read and to write symbolic number sentences. As well as comprehending the meaning of each numeral in sentences such as 74 + 92 = **r** or 74 + **r** = 92, students need to comprehend the 'syntax' or 'grammar' of each and the number relationship underpinning each. Students who have mathematics learning disabilities have difficulty implementing these specific aspects of the information. (Cawley & Reines, 1996; Ginsburg, 1997; Hasselbring, Goin, & Bransford, 1988; McCloskey & Macaruso, 1995).

A third aspect relates to students' ability to recognise and comprehend order among numbers. For the task 74 + **r** = 92, students use this aspect to recognise that 74 is less than 92 (and therefore the task ‘makes sense’ or is plausible in terms of whole numbers) and that 20 is an approximate value for q because it lies between them. A knowledge of place value can assist them in making this decision; nine tens is higher than seven tens. Ordinal awareness of numbers also allows students to recall, if necessary, other numbers in the vicinity of a number, for example, that 90 is close to 92.

A fourth aspect relates to the ability to perform mental computations. Students with a well developed information processing skills for a domain of numbers are more able to do this. Other students learn the arithmetic concepts but have difficulty using them efficiently; they have difficulty converting their conceptual knowledge of a topic in arithmetic to procedures or actions they can apply to the information. There are two aspects of this; they need to decide the appropriate procedure and to apply it correctly. Students in the course of acquiring this skill are likely to use it in an attention-demanding way. As their competence increases, it is predicted they can do this more automatically. The present investigation asks students to perform a comparatively simple computation; to increment or decrement by one through a range of numbers.

The present study examines various aspects of arithmetic information processing for whole numbers by third and fifth grade students. Its focus is on students’ ability to (1) read numbers and number sentences, (2) comprehend sequence among numbers (3) perform arithmetic procedures. Students need to display these abilities for increasingly complex whole numbers and with increasing proficiency so that they can move seamlessly between the aspects.

Some children do not acquire these information processing skills at the same rate as their peers. At least three cognitive processes can lead to mathematics learning disabilities: (1) a long term memory difficulty in storing and retrieving arithmetic knowledge (Cawley & Reines, 1996; Woodward & Howard, 1994); (2) a difficulty encoding symbolic numerical information in working memory ((Hasselbring et al., 1988; Pellegrino & Goldman, 1987); and (3) slower visual symbolic naming speed that disrupts the acquisition of symbolic patterns and reduces the quality of the symbolic codes in memory (Allor, 2002; Lovett, Steinbach & Frijters, 2000).

For the present tasks, slower naming of arithmetic symbols is expected to influence the number of tasks a student can complete in a given time. It may also mean that a student needs to review the same information more than once to understand it. A long term memory difficulty is expected to influence accuracy in task completion; the student will not have stored in memory the relevant knowledge to complete the task. These two processes are likely to restrict a student’s ability to encode symbolic numerical information in working memory; for efficient learning and problem solving the student needs to have the knowledge necessary for naming, interpreting and relating the information.

Learning the various information processing skills is not sufficient for mathematics performance. Students also need to apply them to numbers of increasing complexity. The present study investigates the influence of number complexity by examining how well students process numerals that have an increasing number of digits (or places). The trend from two to five digit numerals, for example, is one of increasing numerical information load. Investigating how the increase in load influences accuracy for a particular task or procedure provides an indication of the automaticity with which the students process the information.

The present study predicts that numerical information load influences arithmetic information processing for students who are acquiring arithmetic knowledge and that, as students become more proficient, its influence will decrease. It examines these predictions for three arithmetic information processing; reading isolated numbers, recognising ordinal sequence and applying an arithmetic procedure. In terms of mathematics underachievement, it is predicted that relatively low arithmetic achievement is paralleled by a less mature information processing for each component.

Method

Design : The study examined four aspects of arithmetic information processing by third and fifth graders: the ability to (1) recognise 1- to 5- digit numerals; (2) read correct and incorrect number sentences comprising 1- and 2- digit numbers having a single arithmetic operation; (3) recognise ordinal sequence among 1- to 3- digit numbers; and (4) increment or decrement by 1 from a specified number. All of the tasks had a fixed time. This permitted the comparison of processing times across students. The mathematics achievement of students at each year level was normally distributed.

Participants : The participants were 73 third graders and 82 fifth graders from four schools in metropolitan Melbourne. All were judged by their teachers to be in the 'average range' for general ability. None displayed learning characteristics likely to lead to learning difficulties. The schools were characterised by medium ratings for educational maintenance allowance and low ratings for family mobility and language background other than English. The participants' teachers reported that on earlier mathematics assessment tasks, each grade of students spanned the range of expected student achievement, from low to high achievers.

Procedure : Tasks 1 to 4 below were administered under controlled time conditions. This permitted an examination of automatic processing for each task.

Task 1. The ability to recognise isolated 1- to 5- digit numerals.

For this task, each student saw rows of four numbers. Each row comprised numbers of 2-. 3-, 4- or 5- digits; this variable was used to examine information load. One of the four numbers repeated a number earlier in the row. The students were asked to identify the repeated number by circling its second appearance. As noted earlier, they could use a range of strategies to make the visual discriminations, depending on their existing knowledge of number symbolism. A sample task is shown in Figure 1.

63	24	47	63

Figure 1 : Sample numeral encoding task.

The set of tasks for each digit range counterbalanced the repeated number in the second, third and fourth positions. The order of rows for each digit range was determined by random selection.

In all, students were exposed to 16 rows. They had forty seconds to do as much as they could. Each correct response was scored 1 point and the maximum possible score was 16.

Task 2. The ability to recognise number sentences that comprised 1- and 2- digit numbers linked by a single arithmetic operation.

This task required students to recognise strings of arithmetic symbols. All strings had ‘arithmetically correct syntax’. Some were correct number sentences and some were incorrect. This was discourage the use of automatised number fact knowledge by some students during selection.

Students’ attention was drawn to a written number sentence (the target). They then saw a row of four number sentences, one of which was the target sentence. Their task was to select and circle it.

They were introduced to the task as follows: “ Several children were asked to write down some number sentences. Peter's number sentences are written below on the left hand side. His number sentences, along with Ann’s, Tom’s and Jean’s, are written on the right hand side. They are mixed up. Try to find each of Peter’s sentences in the set on the right hand side. As soon as you find each one, circle it. Work as fast as you can.” A sample task is shown in Figure 2.

	What Peter wrote	Find here what Peter wrote and circle it			
a	6 + 5 = 8	4 + 5 = 9	4 + 3 = 6	6 + 5 = 8	5 + 4 = 7

Figure 2 : Sample number sentence recognition task.

In all, there were 28 rows and students had 60 seconds to do as many as possible. The rows included the four arithmetic operations. One of the four items in each row had the same digits and arithmetic operation as the target sentence, but in an alternative order.

In seven of the rows the target number sentence was correct. The order of rows was determined by random selection.

Task 3. The ability to recognise sequence among numbers

This task required students to decide whether a target number was one of the set of counting numbers between two other numbers (the terminal numbers). An example of the task is shown in Figure 3.

5	9
6	

Figure 3 : Sample awareness of ordinal sequence task.

In this case the target number is 6. The students needs to decide whether it lies in the set of counting numbers between 5 and 9. The instruction given to students is:

“In each of the following boxes you can see two numbers on the line and one number under it. When you count from one number on the line to the other number, will you say the number underneath ? Does the number underneath come between the two numbers on the line when you count from one to the other. If the number underneath does come between the two line numbers when you count, circle y (for yes). If it doesn't come between the two line numbers when you count, circle n (for no). “

There were 48 instances of this task and students had 2 minutes to work on it. Of these, 16 each had 1-digit, 2-digit and 3-digit numbers. For each type of number, 8 involved an increasing sequence and 8 a decreasing sequence, 8 were true and 8 were false. Each correct response was scored 1 point and the maximum possible score was 48.

Students completed the sets in the order 1-digit, 2-digit and finally 3-digit numbers. Within each set of numbers the 16 stimuli were arranged in a random order of administration.

Task 4. The ability to count up or down by 1 from a specified number.

This task required students to increment or decrement by one step from a specified number, for 2-, 3- and 4- digit numbers. For incrementing tasks, the specified number was 1 unit less than a multiple of 10 or 100 and for decrementing tasks the specified number was a multiple of 10 or 100. Each instance of the task was written and students wrote their response, for example : (1) Write the number one more than 4099 ______ and (2) Write the number one less than 310______.

In all, students were exposed to 12 instances of this task; 4 for each of 2-, 3- and 4-digit numbers, with 2 incrementing and 2 decrementing for each type of number and had 1 minute to work on it. Each correct response was scored 1 point and the maximum possible score was 48. The instances were administered in a pre-determined random order.

Task 5. Whole number knowledge test

Students' knowledge of whole numbers was assessed by compiling a set of tasks that included

(1) whole numbers computation tasks covering the use of the addition, subtraction, multiplication and division operations with 1-, 2 and 3- numbers, the solution of equations and the estimation of arithmetic outcomes (16 tasks).

(2) word problems, each based on one of the four operations (8 tasks).

The set of tasks was designed using Levels 2 to 4 the Number Strand from the Key Learning Area of Mathematics in the Curriculum & Standards Framework 11 (CSF11) (Board of Studies, 2000). This is the guiding curriculum for Victorian schools. An initial set of a possible tasks was submitted to a panel of twelve experienced primary school teachers, who each ranked in order the items they believed assessed conventional mathematics knowledge in Grades 3 and 5.

The set of tasks above was compiled from this. The tasks were arranged in an approximate order of difficulty. Overlap between items was minimized.

The test was administered in a written form on a group basis. It was untimed. One mark was allocated for each correct response. The maximum score was 24. The descriptive statistics for the test at each year level are shown in Table 1.

Table 1 : Descriptive statistics for the whole numbers test at each year level.

	Grade 3	Grade 5
Mean	14.6	18.7
Std. Deviation	5.6	4.1
Skewness	-.31	-1.27
Kurtosis	-.54	2.02
Minimum score	3	7
Maximum score	24	24

Total scores on this test were used as an estimate of whole number computation skill knowledge. As well, the names of the students who achieved in the lowest quartile for each grade were cross- referenced with objective teacher ratings to identify those students they judged to be 'at risk' of mathematics learning difficulties. This included those currently showing low achievement. A second group, those in the 25^{th} to 75^{th} percentile range were cross- referenced with objective teacher ratings to identify those judged to be 'average progress' mathematics learners.

Results

The performance of the average progress and at risk students at each year level on each task are reported in this section. General linear modelling procedures were used to examine the influence of grade level, category of mathematics achievement and numeral range on performance. Predicted differences were examined using planned comparisons. The extent of association between each area of information processing and computational performance was examined using Pearson's product moment correlation coefficient.

Recognising numbers The mean number recognition performance (Task 1) for each type of number (2, 3, 4 and 5 digits) by average progress and at risk students at each grade level is shown in Table 2.

Table 2 : Mean numeral encoding performance at each grade level (maximum score =16).

Numeral encoded	Grade 3		Grade 5	
	At risk N= 23	Average progress N= 50	At risk N= 28	Average progress N= 54
2 digit	12.78	14.32	14.46	16.00
3 digit	8.73	10.16	12.59	14.61
4 digit	4.31	7.47	8.93	14.32
5 digit	4.19	6.84	7.75	12.98

Increasing numerical complexity, by adding a place, influenced students' recognition ability at both grade levels ($p < .01$). Grade 3 students recognised 2- and 3-digit numbers more easily than 4-digit numbers and 5-digit numbers ($p < .01$) were most difficult. Grade 5 students recognised 2-, 3- and 4-digit numbers with similar efficiency and found 5 digit numbers more difficult ($p < .01$).

Total recognition ability correlated with students' whole number computation skill at both year levels (Grade 3, $r = .43$, $p < .05$; Grade 5, $r = .54$, $p < .01$).

Recognition accuracy for each type of number generally increased with grade level. The two grades recognised 2 digit numbers with equal ease. For the 3- to 5-digit numbers, Grade 5 students achieved a higher score than their Grade 3 peers ($p < .01$).

The average progress and at risk students at each year level differed in their recognition ability. Average progress third graders recognised 2-, 3- and 4- digits more efficiently than their 'at risk' peers ($p < .01$) while average progress fifth graders recognised 3-, 4-

and 5- digits more efficiently than their ‘at risk’ peers ($p < .01$). The lower achieving mathematics students at each year level were less efficient in their numeral encoding than their average progress peers.

Matching number sentences The fifth graders read by recognition the number sentences more efficiently than their third grade peers ($p < .01$). The mean number of sentences read correctly were (1) 16.72 and 20.93 for the third and fifth average progress students respectively and (2) 11.67 and 13.01 for the third and fifth at risk students. Number sentence reading ability correlated with whole number computation skill for both year levels; $r = .42$, $p < .05$ for Grade 3 and $r = .56$, $p < .05$ for Grade 5. At risk students at both year levels read the number sentences less efficiently than their average progress peers ($p < .01$). The lower achieving mathematics students at both year levels were less efficient in their number sentence encoding ability than their average progress peers.

Awareness of ordinal sequence The ability to recognise ordinal sequence (Task 3) for numbers that varied in place value complexity from 1 to 3 digit numbers by average progress and at risk third and fifth graders is shown in Table 3.

Table 3 : Mean ability to recognise ordinal sequence by average progress and at risk students at each year level (maximum score =16 for each type of number).

Recognising ordinal sequence	Grade 3		Grade 5	
	At risk N= 23	Average progress N= 50	At risk N= 28	Average progress N= 54
1 digit numbers	5.94	8.63	10.43	12.94
2 digit numbers	4.07	6.75	8.39	9.46
3 digit numbers	2.81	5.14	7.58	8.02

Numerical complexity influenced sequencing ability at both year levels ($p < .01$). Students levels found it easier to sequence 1-digit numbers than 2-digit numbers ($p < .01$) and 3-digit numbers ($p < .01$) were most difficult. Year 5 students were more efficient than Year 3 students in recognising ordinal sequence for the three types of numbers ($p < .001$). These data are consistent with the prediction that awareness of ordinal sequence increases with grade level.

Computational performance was associated with sequencing ability for third grade level ($p < .01$) (Pearson's correlation coefficients were .39, .57 and .63 for 1, 2 and 3 digit numbers respectively), but not for fifth graders (Pearson's correlation coefficients were .12, .09 and .12 for 1, 2 and 3 digit numbers respectively). These data indicate that the influence of sequencing ability on computational ability decreases at the higher grade level. While some third graders may still be using counting and sequencing to complete numerical computations, fifth graders are less likely to do this.

The average progress and at risk students differed in their efficiency to recognise sequence for all ranges of numbers at both year levels ($p < .01$). These data are consistent with the difficulty some students have in identifying and using sequence among numbers.

Increment / decrement by 1 The ability to count up (or increment) and count down (or decrement) by one (Task 4) for each range of the set of counting numbers is shown in Table 4 .

Table 4 : Mean ability to increment or decrement by one (mean number correct) in each range of the set of counting numbers (maximum score =12 for each range).

Range of numbers	increment		decrement	
	Grade 3 N= 73	Grade 5 N= 82	Grade 3 N= 73	Grade 5 N= 82
hundreds range	2.87	3.76	2.52	3.71
thousands range	2.21	3.34	2.01	3.06
tens of thousands range	1.95	2.95	1.65	2.74

Numerical complexity influenced counting up and down by one for both year levels ($p < .01$). Both third and fifth graders found the 3-digit number range easier than in the 4-digit range ($p < .01$). The 4-digit and 5-digit number ranges did not differ in difficulty for either grade level ($p > .05$). This ability was associated with whole number knowledge for both year levels (Pearson's correlation coefficient for year 3, $r(71) = .62$, $p<.000$ and for year 5, $r(79) = .33$, $p<.01$.)

The at risk students were less able than their grade matched average progress peers to count up or down by one ($p < .01$). The difference between counting up and down was generally greater for the at risk group.

Taken together, these data are consistent with an evolving information processing capacity for whole numbers. The four components were correlated positively with whole number knowledge. As well, numerical information load for three components influenced task performance.

Students in the 'at risk' range at each year level performed less efficiently on the four component tasks. They had greater difficulty reading isolated numbers, reading number sentences, recognising ordinal sequence and using arithmetic procedures. Their performance suggests that they can perform the components but do so more slowly and with less complex numbers. These findings are consistent with the earlier investigations of Bull and Johnston (1997) and McCloskey and Macaruso (1995).

Implications for teaching students who have mathematics learning difficulties.

The results indicate the need to take the four aspects of information processing into account when diagnosing and remediating mathematics learning difficulties. Contemporary mathematics diagnosis focuses on students' ability to manipulate separate aspects of mathematics tasks in attention demanding ways in clinical interview contexts. Their ability to process aspects of the information defining a task, such as naming rapidly each numeral, using the syntax of number sentences or recognising order among numbers, is frequently taken for granted. The present study shows that these abilities correlate with their arithmetic ability, that those who underachieve do these abilities less well and that the complexity of numbers to which these are applied determine the efficiency with which they are done.

It is strongly recommended, therefore, that students' relevant information processing abilities, as well as their mathematics abilities, are assessed during mathematics diagnoses. It is also suggested that attention be given to assisting students to increase their information processing skills during intervention. Components such as encoding individual numerals and number sentences are frequently overlooked in conventional mathematics teaching. Immature knowledge here can lead to mathematics learning difficulties. Neglecting them in intervention may render the intervention less effective.

Teaching programs need to assist students to handle an increasing amount of numerical information. The assumption that once a mathematics procedure has been learnt it can be applied across a range of numbers is ill-founded. Teaching may need to provide the opportunity for students to manipulate each load in an attention demanding way initially and then to gradually automatise it.

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