

Dyscalculia : A unifying concept in understanding mathematics learning disabilities

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Abstract : Individuals display a mathematics disability when their performance on standardized calculation tests or on numerical reasoning tasks is comparatively low, given their age, education and intellectual reasoning ability. Low performance due to cerebral trauma is called acquired dyscalculia. Mathematical learning difficulties with similar features but without evidence of cerebral trauma are referred to as developmental dyscalculia. This review identifies types of developmental dyscalculia, the neuropsychological processes that are linked with them and procedures for identifying dyscalculia.

The concept of dyslexia is one with which professionals working in the areas of special education, learning disabilities are reasonably familiar. The concept of dyscalculia, on the other hand, is less well known. This article describes this condition and examines its implications for understanding mathematics learning disabilities.

Individuals display a mathematics disability when their performance on standardized calculation tests or on numerical reasoning tasks is significantly depressed, given their age, education and intellectual reasoning ability (Mental Disorders IV (DSM IV)). When this loss of ability to calculate is due to cerebral trauma, the condition is called acalculia or acquired dyscalculia. Mathematical learning difficulties that share features with acquired dyscalculia but without evidence of cerebral trauma are referred to as developmental dyscalculia (Hughes, Kolstad & Briggs, 1994). The focus of this review is on developmental dyscalculia (DD).

Students who show DD have difficulty recalling number facts and completing numerical calculations. They also show chronic difficulties with numerical processing skills such recognizing number symbols, writing numbers or naming written numerals and applying procedures correctly (Gordon, 1992). They may have low self efficacy and selective attentional difficulties (Gross Tsur, Auerbach, Manor & Shalev, 1996).

Not all students who display low mathematics achievement have DD. Mathematics underachievement can be due to a range of causes, for example, lack of motivation or interest in learning mathematics, low self efficacy, high anxiety, inappropriate earlier teaching or poor school attendance. It can also be due to generalised poor learning capacity, immature general ability, severe language disorders or sensory processing.

Underachievement due to DD has a neuropsychological foundation. The students lack particular cognitive or information processing strategies necessary for acquiring and using arithmetic knowledge. They can learn successfully in most contexts and have relevant general language and sensory processing. They also have access to a curriculum from which their peers learn successfully.

It is also necessary to clarify the relationship between DD and reading disabilities. Some aspects of both literacy and arithmetic learning draw on the same cognitive processes. Both, for example,

¹ This article was published in Australian Journal of Learning Disabilities, 2003 8, (4).

demand the ability to learn and use alphanumeric symbols and to retain these in memory. It is possible that the memory processes involved in learning letter clusters are those also used to learn arithmetic symbolism (Geary, 2001). The rapid retrieval of abstract knowledge from long term memory (such as under the condition of rapid automatised naming) is also likely to be shared both by literacy and arithmetic learning (Bull & Johnson, 1997).

This led investigators and diagnosticians to distinguish between primary and secondary DD. Levin, Goldstein and Spiers (1997) noted that some individuals can read words but not numbers while others show DD as a result of more general spatial processing difficulties, leading to a spatial disorganization for numbers,

Students whose only learning disability is DD have been reported in several studies (for example, Ozols & Rourke, 1991). They comprise 3% to 6.5% of the general school population (Gross Tsur, Manor & Shalev, 1996; Von Aster, 1994), lie within the average intelligence range and come equally from both genders. DD is a problem that needs to be targeted by appropriate educational service provision. Its identification is complicated because it can arise either as a specific entity or as part of a broader range of difficulties. It demands appropriate diagnostic procedures and educational interventions.

Types of dyscalculia In order to discuss the types of dyscalculia that have been reported, it is useful to examine the areas of activity in which individuals completing an arithmetic task need to engage. As a first approximation, pupils need to manipulate the numerical information defining the task in the following ways; they need to

- (1) read the data defining the task; this includes naming correctly each arithmetic symbol, including multi digit numbers, comprehending or retrieving its meaning, combining the meanings in the intended ways and discriminating relevant from irrelevant data,
- (2) decide what the acceptable outcome will be like,
- (3) link the task with earlier learning,
- (4) recall and apply appropriate procedures to the data given,
- (5) recall particular number facts,
- (6) manage, plan, monitor and evaluate the effectiveness of their efforts, and if these are judged to have been unsuccessful, to re work the task.

These activities are not necessarily used in a uni directional way in the sequence shown. Individuals can perform more than one simultaneously and can move between two or more in a reciprocal way. The sequence assists in identifying the types of thinking that may be implicated in developmental dyslexia.

In a landmark article, Kosc (1974) identified six types of DD. Subsequent investigators, for example, Rosselli and Ardila (1997), have validated them. These types are:

- a difficulty using mathematical concepts in oral language, talking about mathematical relationships sensibly (verbal dyscalculia, Kosc (1974); aphasic acalculia, Rosselli & Ardila (1997)). Kosc noted two aspects of this type of dyscalculia : a difficulty (1) identifying spoken numerals (although the individuals could read the numerals, and (2) recalling the name of a quantity (although they could read and write the number).
- difficulty manipulating concrete materials, or enumerating a quantity. The difficulty here seemed to involve converting one's arithmetic knowledge to actions or procedures in relation to quantities (practognostic dyscalculia, Kosc (1974); spatial acalculia, Rosselli & Ardila (1997)).

- a difficulty reading mathematics symbols such as numerals (lexical dyscalculia, Kosc (1974); alexic acalculia, Rosselli & Ardila (1997)). Students with this difficulty can talk about mathematics ideas and comprehend them in oral discussion but have difficulty reading both individual symbols and number sentences.
- a difficulty writing mathematics symbols: (graphical dyscalculia, Kosc (1974); agraphic acalculia, Rosselli & Ardila (1997)). Students can comprehend mathematics ideas in oral discussion and can read numerical information but have difficulty writing their understanding in maths symbolism.
- a difficulty understanding maths ideas and relationships; (ideognostic dyscalculia, Kosc (1974); anarithmetia, Rosselli & Ardila (1997)).
- a difficulty performing specified mathematical operations; (operational dyscalculia, Kosc (1974); frontal acalculia, Rosselli & Ardila (1997)).

Any one student does not necessarily show all areas of difficulty. Any of the six types may occur, either in isolation or in combination.

Kosc's model was based on arithmetic outcomes. More recent approaches identify the information processing needed to obtain these outcomes. Macaruso, Harley and McCloskey (1992) and Temple (1992) developed cognitive models of number processing and calculating. They identified three areas in which DD students may show difficulties;

- disorders of number processing; difficulty reading and comprehending arithmetic symbols (operational symbol processing). This matched Kosc's lexical and graphical dyscalculias.
- disorders in establishing arithmetic facts; difficulty learning, automatising and recalling arithmetic facts. This synthesised Kosc's verbal, ideognostic and practognostic dyscalculias and added an explicit memory component. Some students learn arithmetic but have difficulty recalling it, due to a difficulty either in memory encoding or retrieval.
- disorders of arithmetical procedures; this refers to a difficulty calculating. It matches Kosc's operational dyscalculia.

Impaired information processing of arithmetic can lead to various performance patterns (McCloskey & Caramazza; 1987): students may differ in how they

- comprehend as opposed to express numerical information,
- process numbers written in numerals rather than in words ,
- understand individual digits in written numbers as opposed to the place of each digit (the lexical syntactic distinction) and
- handle spoken as opposed to written information demands (the phonological/graphemic distinction).

The performance of DD students support the model. Some can apply algorithms correctly only when provided with the relevant written number facts (Kaufmann, 2002; Temple, 1991). Some have accurate number processing skills but lack other components of the model. Some may recall number facts accurately but have difficulty using calculation procedures (procedural dyscalculia) (McNeil & Burgess, 2002). Some can comprehend and produce numbers but have difficulty recalling numerical facts or doing simple arithmetic computations (Shalev, Weirtman & Amir, 1988).

Areas of processing that typify developmental dyscalculia

Semantic versus rote symbolic processing Many students with DD can learn the quantitative aspects of a mathematics concept better than the symbolic aspects. Some differ in their understanding of the analogue properties of numbers, such as the comparative size of a number versus the symbolic properties of numbers, such as the equality of different forms of a number

(Polk, Reed, Keenan, Hogarth & Anderson, 2001). They may find tasks that require comparing the size of numbers easier than tasks that require symbolic number knowledge, such as comprehending arithmetic symbols. These findings suggest that symbolic number knowledge and magnitude representations are functionally independent and separate aspects of number knowledge.

Arabic number reading and phonological recoding Being able to subvocalise written numbers, that is, to tell yourself what they say, is a key aspect of using mathematics symbolism. This is a mediating factor learning to read and write them. A key aspect of this is being able to say aloud written arithmetic numerals. This is referred to as the ability to transcode numbers and is assessed by having students read numerals aloud or to write dictated numerals.

Children with DD have difficulty with numeral transcoding (Noel & Turconi, 1999; Sullivan, Macaruso & Sokol; 1996). For numerals of more than one digit or place, some DD students can retain each digit but have difficulty retaining its place; a 'syntactic' error, (Sullivan, et. al., 1996). They may, for example, read '236' as "three hundred and sixty two". Others show the alternative difficulty; they retain the syntactic component of the number but make lexical errors, naming the digits incorrectly ('digit dyslexia', Temple, 1991). They may, for example, read '236' as "five hundred and twenty seven". These students often have difficulty reading aloud number words only; they can write dictated numbers, recognise and comprehend numbers and do some calculations (Macoir, Audet & Breton, 1999).

The findings suggest that development dyscalculia involves specific categories of disorder. Reading numeral words is more difficult than arabic numbers but the nature of the errors is similar. There are separate pathways for reading numbers written in words, reading numbers written in symbols and for writing spoken numbers (Macoir, et al., 1999). The reading deficit coexists with good phonological reading skills.

Spatial disorders are frequently linked with written calculation difficulties (Hartje, 1987) and cause numeral transcoding problems. The spatial difficulty can be general or restricted to arithmetic and is referred to as 'spatial dyscalculia' (Rosselli & Ardila, 1997).

The findings of the separate pathways for reading and writing numerals have implications for diagnosis and teaching. Some students may need to learn each pathway separately and then to integrate them. Some may need to learn the self talk necessary for converting the spatial information in multi digit numbers into self scripts to help them to interpret the numbers.

Sequencing difficulties Being able to link pieces of information in a sequence or to move through the sequence to identify the item that 'comes next' are critical aspects of mathematical learning and thinking. Examples include being able to (1) recognise the link between quantities that have five, six and seven items, (2) learn the names of the places in order and (3) learn a set of arithmetic actions or steps in order. There are two aspects here: (1) learning a sequence of information and (2) using the sequence effectively.

Students with neurological impairments have difficulty generating and producing counting sequences (Lacert, 1997) and applying steps in arithmetic procedures in the correct sequence (Gordon, 1992). When required to work out the number of items in a set, they may have difficulty recalling (1) the number names in order or (2) the items they have already tagged with a number name. They may have difficulty recognising which of the numbers 7 and 9 refers to the greater amount because they cannot link the two numbers in order. These difficulties often co occur with dysphasia and / or an inability to direct movements in dyspraxia. Children in the early stages of learning to count show similar errors.

Cardinality difficulties A basis of the concept of number is cardinality, an understanding of the number of items in a set, that is, its numerosity. This understanding allows students to comprehend both quantity and number. Students learn that the cardinality of a set is the final number name you say when you have correctly tagged each item with a number name in order.

Being able to sequence and to count correctly is key but insufficient. Some students with DD can count and use ordinal numbers effectively but cannot acquire basic cardinal skills. Ta'ir, Brezner

and Ariel (1997) propose an innate "cardinal/ordinal skills acquisition device" that integrates ordinal and cardinal number knowledge. Students with DD have difficulty making this synthesis. This restricts their ability to interpret and represent quantities or develop a concept of number. To decide the greater of two quantities they see, for example, they are more likely to use perceptual information rather than number logic.

Difficulty recalling number facts. Many DD students have difficulty recalling basic addition, subtraction, multiplication or division number facts automatically (Macaroso et.al., 1992), sometimes even with well developed algebraic and arithmetic conceptual knowledge (Hittmair Delazer, Sailer, & Benke, 1995). Instead, they need to 'reconstruct' the facts continually (Fleischner, Garnett, & Shepherd, 1982) using procedures that demand attention or thinking space, such as counting aloud or using their fingers or strokes on a page to work them out (Gray & Tall, 1994). These fact retrieval difficulties are frequently resistant to mathematics teaching and persist over primary education (Ostad, 1999).

The reasons for these difficulties is not clear. The conditions under which students usually automatize number facts include repeated use (that is, 'drill and practice') and an awareness of meaningful base on which a set of related facts is based. They may learn the eight times table by using the 'go up 10 then down 2' pattern. They use the regularities or consistent patterns to link facts from several tables. To learn each fact they need to 'decontextualise' or 'abstract' it. This involves removing the meaningful support for it.

Some investigators suggest that the basic facts for each of the four arithmetic domains are stored in memory in a way that matches a two-dimensional network. Each fact has three numbers; the initial, change and outcome numbers. In this model, the set of initial numbers is arranged along one dimension and the set of change numbers along the second (Ashcraft, 1992). The outcome or 'answer' for each fact is at the intersection of the initial and change numbers. An example is the grid for showing the multiplication facts in Figure 1.

		Initial numbers							
		x	1	2	3	4	5	6	7
Change numbers	1	1	2	3	4	5	6	7	
	2	2	4	6	8	10	12	14	
	3	3	6	9	12	15	18	21	
	4	4	8	12	16	20	24	28	
	5	5	10	15	20	25	30	35	
	6	6	12	18	24	30	36	42	
	7	7	14	21	28	35	42	48	
	8	8	16	24	32	40	48	56	

Figure 1 : A schematic representation of the multiplication facts in long term memory.

Each network is learnt gradually. During the learning, the types of errors both able and DD students make recalling facts suggest the aspects of each network they have in place. For multiplication facts, the error is often another number for that table (Campbell, 1987); for $7 \times 9 =$, they may answer 56 or 72. This interference from related facts suggests that the entire table, as well as the specific fact, are stimulated in the student's memory (Ashcraft, 1992). Interference also occurs between domains, for example, for ' $3 + 4 =$ ', a frequent response is "12". When given time, students can frequently self correct. The errors suggest a difficulty suppressing related facts (Geary, 2001).

Some students have difficulty forming these networks. Their difficulty is with learning each fact. To do this, they need to link the numbers and operation at the same time (Geary & Brown, 1991) in their working memory or thinking space. Working memory efficiency depends on the information to be handled. Learners can retain and think about information better when they have the knowledge necessary for interpreting the information and can recall it effectively (Bull et.al., 1997; Ericsson & Kintsh, 1995).

Working memory affects number fact recall (Adams & Hitch, 1998; Geary et.al., 1991; Kaufmann, 2002). Students with DD have a lower digit (or memory) span than average learners (Geary & Brown, 1991). This shorter span is linked with the slower recall of number names during counting (Hitch & McAuley, 1991) and less practice with arithmetic ideas due to selective avoidance. The slower naming of information makes rehearsal of the knowledge more difficult. Students who name parts of a number fact more slowly will be less able to retain the fact as a whole in working memory.

Difficulty automatizing arithmetic facts, then, could be due to a general deficit in the speed of processing information and retrieving knowledge from long term memory (Ackerman & Dykman, 1995; Bull & Johnston; 1997). Students who had difficulty recalling number facts are slower in speeded counting, speed of speech and on some measures of speed of access (Temple & Sherwood, 2002).

Some can recall automatically the number facts for some arithmetic domains only, (McCloskey, 1992; van Harskamp, Rudge & Cipolotti, 2002). Recall difficulties can also be restricted to arithmetic knowledge (Kaufmann, 2002); the learners can retrieve non arithmetic knowledge. The results support a modular, specialized system for storing and retrieving arithmetical facts.

Given that the recall of facts requires students to say each fact aloud or subvocally during learning, the issue of whether arithmetic facts such as $7 \times 8 = 56$ are stored in memory in a phonological or sound based form has frequently been raised. Whalen, McCloskey, Lindemann, & Bouton (2002) suggest that this is not so, showing that individuals can retrieve them when unable to say either the arithmetic problem or its outcome.

Behavioral and attentional aspects of developmental dyscalculia In order to learn arithmetic concepts, students need to have sufficiently high self efficacy as learners of arithmetic and be able to invest attention effectively in the learning activity. Executive functioning is a link between attention and arithmetic. Selective attentional difficulties co occurs with the dyscalculia subtypes (Lindsay, Tomazic, Levine & Accardo, 2001).

Students with DD show lower self efficacy (Gross Tsur, Auerbach, Manor & Shalev, 1996) and a higher incidence of behavioral problems than average learning peers (Shalev, Auerbach & Gross Tsur, 1995). Students with both dyscalculia and dyslexia exhibit more severe attentional and emotional difficulties impairments (for example. externalizing types of problems) than do those without learning disabilities in both areas.

Neuropsychological correlates of developmental dyscalculia

Completing an arithmetic task or learning new mathematical knowledge draws on a range of neuropsychological processes. We noted earlier some of the key ways in which pupils need to manipulate the numerical information to solve a task. Each step draws on several areas of neuropsychological processing. To illustrate the complexity of this, examine the processing involved in the first step, that is, to read and comprehend the numerical data defining the task. Students need to

- learn and store the visual configuration for each symbol and the use of the spatial order of digits in a multi digit number.
- learn and store the spoken name for each symbol; for numbers greater than ten this involves learning to sequence two component names and to retain each in short term memory.
- learn and store the quantitative form of each symbol,
- link the visual, spoken and quantitative forms for each symbol, retrieve them fast enough to retain each aspect of knowledge in short term thinking space and move between them.
- learn how to generate new symbols and their meanings from existing knowledge.
- link a string of symbols with a set of meanings, convert a string to a spoken form.

Each of these requires adequate information processing in particular parts of the brain. For any student, some areas operate more effectively than others.

Reasons for examining neuropsychological correlates of developmental dyscalculia Each area of the brain processes more arithmetic information. DD can co occur with difficulties in attention and memory, auditory and visual memory, attention, developmental dyspraxia, developmental dysphasia, dyslexia and autism and childhood psychoses. By identifying the co occurring non mathematical difficulties, you can infer the likely cause of a particular case of DD.

Behaviours that may otherwise seem unrelated and independent can be seen as linked when one is aware that they are mediated by the same cortical processes. Teachers and diagnosticians can use behavioural checklists to collate and synthesise the range of abilities / behaviours individual students show and to understand the nature of an incident of DD.

Developmental dyscalculia and cerebral locations Developmental abnormalities in both cerebral hemispheres can lead to DD (O'Hare, Brown & Aitken, 1991). Right hemispheric dysfunction leads to difficulties understanding the properties of quantities, spatial learning problems (for example, understanding and using place value) and using arithmetic knowledge to solve real life problems. Left hemispheric dysfunction leads to difficulty comprehending the abstract meanings of numbers, sequencing numerically and maths operations. The abnormalities can be due to (1) conditions such as epilepsy, (2) abnormal development, (3) maturational lag, (4) damage to late myelinating neural tissue, or (5) lateralization disorders.

Left and right hemispheric dysfunction occur with approximately equal frequency in primary students with DD (Shalev, Manor, Amir & Wertman Elad, 1995). Left hemispheric dysfunction leads to greater arithmetic difficulties.

Right hemispheric immaturity One type of DD has been linked with nonverbal reasoning disabilities. Characteristics of Developmental right hemisphere deficit syndrome (DRHS) (Gross Tsur, Shalev & Manor & Orly, 1995) include :

- strong verbal, reading and spelling skills with inadequate social interactional and paralinguistic abilities,
- impaired visuospatial skills, verbal IQ greater than performance IQ, severe graphomotor problems,
- neurological "soft signs" of right cerebral dysfunction, neurological indicators on the left side of the body,
- dyscalculia and
- emotional and interpersonal difficulties;

Students whose learning disability is restricted to arithmetic frequently show higher verbal than visual organisational reasoning ability (Silver, Pennett, Black, Fair & Balise, 1999).

Left hemispheric dysfunction : The angular gyrus and calculating Left hemispheric dysfunction includes right side soft neurological signs, performance IQ less than verbal IQ, dyslexia and intact visuo spatial functions (Shalev, Manor, Amir & Wertman, 1995). Difficulties in one key area, the angular gyrus in the left parieto occipital region, leads to the sequential processing deficit noted earlier (Davis, Bryson & Hoy, 1992) and restricts calculation processes (Duffau, Denvil, Lopes, Gasparini, Cohen, Capelle & van Effenterre, 2002). It has separate aspects for multiplication and subtraction calculations. It is not critical for the recall of multiplication facts (van Harskamp, Rudge & Cipolotti, 2002).

The influence of this area on arithmetic calculations and learning number facts is not surprising. We have already noted that, in order to learn these, students need to implement a range of relevant activities, for example, retain the name of each symbol efficiently, possibly use counting to interpret arithmetic operations. The angular gyrus handles functions such as linguistic information processing, naming, maintaining serial order, visual analytic skills and language comprehension.

The characteristic indicators of difficulties in this area, sometimes referred to as ‘developmental Gerstmann's syndrome’ (PeBenito, Fisch & Fisch, 1988) are as follows:

- finger agnosia, observed using finger differentiation or naming tasks,
- agraphia or dysgraphia, observed in poor handwriting, difficulty copying writing from blackboard and poor spelling,
- right left disorientation, shown in directional confusions,
- dyscalculia,
- constructional dyspraxia and
- emotional problems.

Some students have Gerstmann difficulties and also have difficulty with recalling the names of items, auditory processing and reading and spelling (Davis, Bryson & Hoy, 1992). Teachers and diagnosticians can use these to understand the learning characteristics of students with difficulties in this area.

Identifying dyscalculia

Identifying developmental dyscalculic students To identify students with DD, investigators have used general ability measures and tasks that assess number comprehension, production and calculation. An example of the number assessment batteries used is the set of tasks (McCloskey, Aliminosa & Macaruso 1991; Shalev, Orly, Kerem & Ayali, 2001) shown in Table 1.

Table 1 : An example of the number assessment batteries used to identify students with DD.

Areas sampled by each subtest	Examples of tasks
Number processing; <ul style="list-style-type: none"> • comprehend and produce numbers • use lexical data • use syntactic elements of numbers, and skills needed to work with both arabic and lexical numbers 	<ul style="list-style-type: none"> • match written arabic numerals to quantities from 3 to 12. • comprehend quantities by selecting the larger, smaller or equivalent quantity for a given quantity. • select the smaller / larger of two numbers. • order numbers serially. • count aloud the numbers of items in a group. • copy and read numbers, between 1 and 4 digits long, and write them to dictation.
Recall of number facts ; <ul style="list-style-type: none"> • correct use of arithmetic signs • memorizing arithmetic tables 	Recall addition, subtraction, multiplication, and division facts
Arithmetic procedural knowledge; use algorithms to add, subtract, multiply and divide.	Complete complex 2 and 3 digit addition, subtraction, multiplication, and division, decimal and fractions tasks

This table shows the range of tasks used to assess developmental dyscalculia. Teachers and diagnosticians can assess students’ knowledge in each of these areas and assemble a developmental dyscalculia profile for each student.

It has already been noted that some of the types of difficulty characteristic of DD are displayed in as part of the regular development of mathematics knowledge. Indeed, it is possible that some components of DD may be due, at least in part, to the premature cessation of development in these areas.

Procedures used to monitor trends in mathematics knowledge acquisition may be of relevance in diagnosing DD. Identifying those aspects of developmental strands that a person with DD has in place would provide a more accurate description of what an individual with DD does know about mathematics.

The use of general ability measures to diagnose arithmetic disabilities. Patterns of performance on general ability scales such as the Wechsler Intelligence Scale for Children III have been proposed as tools for identifying specific learning disabilities. Many of the studies of WISC type patterns have been unsuccessful in linking unique profiles with particular types of learning difficulty. The earlier observation that DD can be attributed to multiple neuropsychological causes suggests that it is unlikely that DD has a unique WISC type profile. As noted, the core deficits that contribute to DD can include either of visual spatial organizational dysfunction or sequential dysfunction (Branch, Cohen, & Hynd, 1995; Rourke, 1993).

D'Angiulli & Siegel, (2003) compared the WISC profiles of average achieving students (N=121) with students who had a specific arithmetic disability (N=100) and a reading disability (N=143), all in age range 6 to 16 years. The learning disabled students had lower scores than the average achieving students on all verbal subtests. Some in all categories showed a difference between Verbal and Performance IQ scores. The results suggest that the profile patterns on general ability measures are not sufficiently reliable to diagnose learning disabilities in individual students and that detection using patterns of achievement tasks may be more useful.

The stability of academic subtyping over time has also been examined (Silver, Pennett, Black, Fair & Balise, 1999). A group of 80 children aged 9 to 13 indicated four subtypes of arithmetic disabilities; students with learning disability in (1) arithmetic only, (2) arithmetic and reading, (3) arithmetic and spelling and (4) arithmetic, reading, and spelling. Approximately half of the sample 19 months later continued to have DD. Those with learning disabilities in all areas showed the greatest stability. One third of the children with the other subtypes, including those with isolated arithmetic deficits, retained their subtype. The influence of intervening remediation in the investigation is difficult to identify; some students improved in arithmetic knowledge with it, some improved without it and similarly, some did not improve with it. The investigators caution against assuming stability in subtyping, using either general ability or academic tasks. They suggest that an isolated arithmetic deficit may not have a stable neuropsychological profile.

Conclusion

The analysis of mathematics underachievement from a neuropsychological perspective provides an insight into the cause of the learning difficulty and its remediation that is not available from other sources of information. It allows us to see patterns and consistencies in students' performance that would otherwise seem disparate. It is likely that in the future cognitive, educational and neuropsychological perspectives will merge, leading to more a integrated understanding of mathematics underachievement.

As this review suggests, there is at present little overlap between the study of developmental dyscalculia and maths education. Obviously the two areas would share knowledge. It has already been noted that some of the types of difficulty characteristic of DD are displayed in as part of the regular development of mathematics knowledge.

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