The Mathematics Learning Centre : A Model for Intensive Mathematics Assistance

John Munro, University of Melbourne, December 1991

This paper was an invited paper by the Board of Education, Victoria in 1991.

The Mathematics Learning Centre : A Model for Intensive Mathematics Assistance

John Munro, University of Melbourne, December 1991

The approach to intensive strategies in mathematics described in this paper is based on my experiences in developing the Mathematics Learning Centre, initially as part of the former Melbourne College of Advanced Education and later at Victoria College.

From its beginning, in 1978 the Mathematics Learning Centre (MLC) program had three related functions:

1) to assist students also were unable to benefit from regular classroom teaching and their parents,

2) to provide teachers with the opportunity to develop and enhance their knowledge of mathematics underachievement and strategies for assisting mathematics underachievers, and

3) to provide a context in which the learning characteristics of mathematics underachievers could be researched and a range of curriculum modification strategies could be developed, trialled and validated.

Each of these functions has been pursued in a number of ways. In this paper I will discuss:

i) the guiding model of mathematics learning on which the MLC is based,
ii) the participants in the MLC,
iii) the procedures used within the MLC, and
iv) evaluations of its effectiveness.

1. The Guiding Model of Mathematics Learning.

Any guiding model is a function of the perceptions and biases of the individuals developing that model. The model that has provided the rationale for the MLC programme reflects this model; first hand experiences working with students and teachers, trialling, evaluating and modifying teaching procedures and the pursuit of various questions, have modified gradually the developing knowledge base with respect to mathematics learning difficulties and the ways in which problems and issues in this area were consequently perceived. In passing, it has been seen as important to communicate this perception to the teaches involved; the notion of a developing knowledge base that draws on personal experiences and the experiences and findings of others, that is allowed to change in identifiable directions and that, at any time, includes unanswered questions or unresolved issues is reflected in the practical activities undertaken.

1.1 How are Mathematical Ideas Learnt?

The model assumes that students are expected, as an aspect of their on-going education, to acquire an integrated body of mathematics knowledge. The following components can be identified:

i) a knowledge of mathematics content: this includes a knowledge of mathematical elements, procedures, relationships and the use of formal mathematical symbolism, etc. Educators have identified the conceptual and procedural aspects of mathematics knowledge (Hiebert, 1986, Slesmck, 1982) their interrelationships (Nesher, 1986) and the automatization of parts of this knowledge;

ii) a knowledge of mathematics learning strategies, that is, how to learn mathematics; this knowledge includes the actions that students may initiate in order to encode mathematical knowledge. This component is discussed more fully in a subsequent section;

iii) mathematical thinking; this refers to a particular approach to manipulating mathematical ideas and solving problems that is based on mathematical operations (Burton, 1984);

iv) attitudes towards mathematics, how it is learnt and themselves as mathematics learners; the effective component of mathematics learning has been discussed more fully in Munro (1983). A major aspect of this component for mathematics underachievers, mathematics anxiety and the student's self-concept as a mathematics learner, has been discussed more fully in Munro (1987).

These components do not operate independently but rather interact within an integrated and co-ordinated structure such that the mathematics learning strategies to which one has access, and the attitudes held determine in part the knowledge acquired. As well, the perceived success of the learning process by the student is likely to contribute in part to the individual's attitude towards mathematics and the repertoire of learning strategies learned.

In most teaching situations, students are expected to acquire products (2), (3) and (4) above incidentally and spontaneously. As will be discussed later, mathematics underachievers have difficulty acquiring them in this way and need to be assisted to do so.

How are Mathematics Ideas Learnt?

An assumption frequently made by teachers is that students learn by "taking in" what they see, hear or experience in mathematics lessons; mathematics ideas are transmitted by talking, showing, etc. To illustrate the limitation of this assumption, consider how you would interpret the trivial non-mathematical statement "The run is closed" spoken by an official who flagged you down when you were driving along a country road. Your interpretation would be influenced by your perception of the context in which it was spoken, for example, travelling towards a ski run, driving a cattle truck towards a stock-yard, travelling to an athletic meeting. A sprinter driving a cattle truck through snow country may be confused.

In everyday communication situations were are constantly aware of mis-interpreting what we hear and see and needing to modify our initial interpretation. The claim that we learn or comprehend an idea by simply taking it in is not sufficient. The intention of the sentence above may be incomprehensible, partly comprehensible or comprehensible, depending on how we interpret it. We build or assemble our own interpretation that we may subsequently modify.

The Construction of Mathematical Knowledge

The notion of building or constructing an impression of what is perceived is, I believe, also relevant to mathematics learning. Ideas are not transmitted between people. Words, pictures, and actions are transmitted. The meanings associated with these elements of communication are depend on the individuals involved. Some words, sentences, pictures, etc may be comparatively unambiguous in a given situation. Others may be more open to a range of interpretations. I recall working with some fourth grade children on factors. One group worked out that the factors of 24 were 10 or 11 or 14.

They explained as follows "The factors of 24 are 6 and 4 (or 8 and 3) and 6 and 4 make 10, 8 and 3 make 11".

Why Did They add? "In Maths and means add".

Individuals learn by constructing or building in the process of adapting to their environment, using what they already know. They activate their existing knowledge base and modify or add to it in particular ways. This is the constructivist view of mathematics learning, explicated by several mathematics educators (for example, see Cobb, 1986; Kamii 1985; Labinowicz, 1985; Steffe, 1990; Von Glasserfield, 1988). As one might expect, there are various constructivist positions, that broadly fall into two major categories; empiricist-oriented constructivists, and the radical constructivists. They differ particularly in their conception of the relationship between mathematical knowledge and the student and the implications of this relationship for curriculum design and instruction. Empiricist-oriented constructivists see knowledge as existing external to and independent of the student's cognitive activity and the student's task is to construct a representation of this knowledge. Radical constructivists, on the other hand, reject the notion of "out-thereness" of mathematical knowledge; this knowledge is not perceived, intuited or taken in, but instead is abstracted from sensory-motor and conceptual activity by the individual. Those who "see" arithmetical knowledge "out there" are reflecting on structures that they have imposed on reality (Labinowics, 1985);

"We see what we understand, rather than understand what we see (page 23).

Given the purpose of the present paper, I will not enter the current debate, but instead outline the version of the constructivist model on which our programme is based.

Construction involves pupil action and the investment of attention. Learning does not necessarily occur when a pupil is exposed to particular ideas. More than mere exposure is necessary. Pupils learn when they are actively involved in the learning situation, when they have framed up problems or challenges for themselves that they are trying to solve. Learning is seen as a conscious act on the part of the student; the student needs to actively attend to the information presented that is, to invest mental energy or attention in the activity. The focus here is not on what information is presented or on what the teacher does, but rather on what the pupil does with the information presented. This is consistent with the observation that it is the opportunity to learn mathematics that contributes both to achievement and enjoyment of mathematics (Bourke, 1985). Children can learn from a range of teaching experiences, for example, both from concrete experiences and from verbal explanations (Galton, Simon and Croll, 1980; Stigler and Bavaren, 1988) as long as they are afforded the opportunity to learn. The opportunity to construct, using one's general purpose learning strategies and preferred learning styles, etc, needs to be provided.

A Preparedness to Construct

As well as the opportunity to learn, it is important that pupils are motivated to learn; in order to invest attention in an activity the pupils need to have a reason or purpose for attending to the information. In other words, I am assuming that learning is more likely when the pupil has generated or framed up goals for learning. The goals may be to solve the problem at hand, to receive positive regard from the teacher or peers for solving the task, to discover a particular mathematics pattern or a more efficient way for completing a task. The goals that a pupil brings to the mathematics learning task may be only peripherally related to mathematics achievement for example, the pupil's goals may be to complete the tasks more rapidly then peers, to get out to play football as soon as possible. While I have recommended that mathematics ideas the presented as challenges to mathematics underachievers, (Munro), particularly those who display "learned helplessness" or attention deficit disorders, for example, presenting ideas as problems to be solved, encouraging them to make predictions and then to check them, developing mathematical ideas in real-life contexts of interest to the pupil, this is obviously not necessary for all pupils. Many pupils will spontaneously frame up their own goals or problems for a particular learning context. These pupils will feel more confident about imposing their own goals on mathematical rules, etc.

In summary, construction is most likely when pupils have a purpose for constructing. Pupils who don't believe that they can learn, who believe that others don't believe that they can learn, who believe that others don't expect them to learn, or who are not interested in learning are less likely to initiate the construction process.

Construction Strategies

The construction of mathematics knowledge involves conscious action by individual pupils. Pupils differ in how they go about processing mathematics information, the ways in which they allocate attention, etc. I have found it useful to think of the mental activity involved in the construction process in terms of mathematics learning strategies. Mathematics learning strategies are the actions that the pupil uses to construct mathematics knowledge by processing the information presented; in order words, what one does to learn mathematics, or "how to learn" it. There are two types of mathematics learning strategies; mathematics-relevant reasoning strategies and the self-instruction strategies one uses to direct and manage the reasoning strategies. An example of a reasoning strategy involves a pupil making sense of the statement. "3 1/2 = ," by visualizing 3 1/2 as 3 whole pizzas and one half of a pizza, visualizes cutting each pizza into two halves, and then counts the number of halves. An example of a self-instruction strategy would be pupils whenever they are given a statement exemplified by "3 1/2 = ", instruct themselves first to verbalize then to visualize the mixed number as a real-life quantity.

Successful mathematics learning requires pupils to have access to a range of strategies that they use spontaneously and selectively. Students may be expected to differ in the strategies that they employ to construct a particular idea. The range of strategies to which a student has access may be expected to change over time. As with strategy learning in other areas of human performance, regular successful use may lead to a strategy gradually becoming automatized so that the student automatically initiates it, without the need to invest mental energy. When this stage is achieved, the pupil becomes aware of the use of the strategy only when faced with unfamiliar or difficult to process information.

While there are many unresolved issues in relation to mathematics learning strategies (for example, the "blurred" distinction between mathematics-relevant and self-instruction strategies), they provide a useful way of looking at the mathematics learning situation. They draw our attention to the need to distinguish between mathematical knowledge and how we learn it, in much they same way as the area of reading distinguishes between comprehension and the use of comprehending strategies (Whitehead 1986). These strategies are rarely taught directly. Most pupils seem to learn them in much the same way as they learn how to communicate orally, or to read; by incidentally and spontaneously trying out various "thinking action", seeing what words, perhaps guessing at what people around them are doing, and modelling or discussing what others said they did. From this perspective, the reported effectiveness of direct instruction (Bourke, 1985) is not surprising since direct instruction generally requires the pupils to use a restricted range of learning strategies that pupils are heavily cued to use as part of the instruction.

In summary then, the present approach assumes that the construction process requires the use of "construction actions" (or mathematics learning strategies) that are themselves constructed and which most students learn to use spontaneously and incidentally. These strategies are generally learnt co-operatively; the importance of pupils sharing strategies, discussing, modelling and trialling the strategies used by peers, thinking aloud, etc, cannot be over-emphasized.

Successive Approximations

The construction process proceeds via series of successive approximations. At any time in the learning process the pupil constructs an hypothesis about the idea being learnt, trials and tests the representation; the pupil uses feedback to note how well the representation fits reality and then modifies it if necessary. "Errors" have an important place in this learning situation. Pupils learn that the construction process is gradual and they may learn only part of an idea on the first (or each) go. They learn to expect several "bites of the cherry" and to see "errors" not as events to be avoided at all costs, but as signalling the need for further work on the idea. Learning how to do this is important.

The concepts of "success" and "failure" are not relevant within this model of learning. Responses to errors such as "No, you are wrong" are less instructive than "I see what you mean. You're well on the way building the idea". You've handled this part well". With the emphasis on "partial construction" the concept of failure is inappropriate; the pupil is cued to work further. The "You are on the way" statement signals the pupil to re-assess part of the output and perhaps the mental representation at that point. It signals the notion of modification. Pupils can see how then purposes have been partially achieved and can now work on the outstanding aspects. Pupils are cued to ask questions, to trial various actions and to decide whether these work.

In summary, then, my perception of constructivism in mathematics incorporates the following:

1) mathematics ideas are built: new ideas are built on, or added to, existing ones; one's existing knowledge base is gradually modified.

2) learning proceeds via successive approximations that are gradually shaped by the pupil to match the pupil's perception of reality, and

3) pupils differ in the learning strategies to which they have access.

The Characteristics of Students who need Intensive Mathematics Instruction.

Who are the students for whom intensive mathematics instruction is most appropriate? What are their learning characteristics? To examine these questions, we need to look at the ways in which mathematics underachievement has been described and the assumptions that these make about the nature of learning and teaching. Generally, four approaches are discernible in earlier studies (Munro, 1987):

1) the psychological descriptive approach, that has focussed on various psychological difficulties that co-occur with mathematical difficulties,

2) the error analysis approach that has focussed on the types of errors made by students,

3) the neuropsychological approach that has related mathematics underachievement to neurological disorders, through concepts such as "dyscalculia", and

4) the information - processing approach, that has focussed on the relationship between mathematics difficulties and various information processing strategies, such as verbal processing, short-term memory, etc.

The characteristics of each approach, their inter-relationships and the contribution that each makes to an understanding of mathematics underachievement are discussed by Munro (1987).

The four approaches tend to focus on students in isolation rather than on students in relation to their mathematics educational history. Although they generally examine students who have been exposed to formal mathematics instruction, they rarely refer to the characteristics of this instruction. Any formal learning environment makes assumptions about how students learn, that is, makes particular demands on student learning capacities. Learning situations that match more closely how students go about learning are more likely to lead to improvement in mathematics performance. It is not reasonable to assume that all mathematics learning situations make identical demands on how students learn.

The present approach assumes that a student's mathematics performance at any time is a function of the interaction between the student and the learning environment. Mathematics underachievement occurs when there is a significant mis-match between how a student attempts to learn mathematics and the demands made by the learning environment. Transitory mismatches may be expected as a regular component of any learning situation and are resolved, as the student develops new learning strategies and as the learning situation adapts to match student learning styles. The greater the mis-match, the longer it lasts without resolution, the more likely is mathematics underachievement.

Within the group of students who have had chronic difficulty benefiting from the mathematics curriculum that assists their peers, one might identify as a first approximation two sub-groups.

1) those who can be assisted within their regular classroom; these students show on-going mathematics difficulty, but have a reasonable mathematics foundation. Minor curriculum modifications, perhaps assisting the student to maintain on-task attention, to generate goals, to learn more effective problem-solving strategies assist the student to improve.

2) those students who, at one time, are totally "at sea"; they may lack confidence, do not use their general knowledge when learning mathematics, don't know how to go about learning mathematics and present as innumerate. These students at one point in time, cannot easily be assisted within the regular classroom context; they need the opportunity to learn how to learn mathematics, to feel free to experiment and to take risks and to see themselves as able to learn mathematics. This "opportunity to re-learn" can best be provided in an intensive strategies context. In psychometric terms, students in the first sub-groups may achieve at stanines 3 and 4 on normal mathematics tests, while students in the second sub-group would be more likely to achieve at stanines 1 and 2. This interpretation is supported by Pickering, Szaday and Duerdoth's (1988) analysis of special education needs in Victoria.

What proportion of the school population needs access to intensive numeracy instruction on at least one occasion during their educational history? The analysis of the incidence of learning difficulties in Victorian Catholic Schools (Pickering et al, 1988) showed that 13.3% of the school population scored at stanines 1 and 2 on the Progressive Achievement Test in Mathematics (ACER, 1984), while 6.6% of the group scored in this range on two reading tests. The 13.3% of the school population who had severe mathematics difficulties included students who displayed specific mathematic disabilities (2.8%) and students who had intellectual, behavioural emotional or physical disability (10.5%). Whatever the nature of the learning difficulty, approximately one-eighth of students in regular classes may be expected to need intensive mathematics learning disabilities.

While students in the former group may already be involved in integration support programs. Those in the latter group are generally not eligible for this support.

The types of mis-match that lead to mathematics underachievement. Let us look at some of the mis-matches that can occur between student's learning styles and curriculum demands. To do this we will identify frequently occurring assumptions that mathematics curriculum make about student learning styles and the extent to which underachievers can meet these demands.

1) The learning environment assumes that the student can manipulate and process particular types of information in various ways. Some arithmetic disabled students have difficulty:

a) processing quantitative data, for example, enumerating quantities, comparing quantities in magnitude (practognostic dyscalulia; Kosc, 1986), reasoning in terms of conservation in Piagetian-type task (Derr, 1983). The ability to construct mental representations of numbers, such as four, two thousand and six, three quarters, point six, etc., develop from the processing of quantitative data. While most seven year old children can comprehend a two-digit whole number such as twenty three in terms of its cardinal, ordinal and place value properties, mathematics underachievers frequents comprehend it only in a cardinal way, as representing twenty three discrete units.

b) processing visually-presented symbolic data, spatial symbolic data (lexical dyslexia , Kosc, 1986; spatial acalculia, Levin, 1979). These students have frequently not developed strategies for abstracting the intended meanings or ideas in symbolic statements. When they hear the statements read, or learn how to read them (Munro,) their mathematics performance improves. In passing, it should be noted that this is not a visual-processing difficulty per se, but a difficulty converting a symbolically presented message to an alternative language form;

c) processing arithmetic operations mentally (anarithmetria, Hacaen, 1962; operational dyscalculia, Kosc, 1986). Many underachievers have difficulty comprehending the four basic operations by associating mental actions with them, for example, counting down with subtraction. They needs to learn these actions by internally gradually corresponding physical actions (Van Erp & Heshusius, 1986);

d) processing and handling all of the necessary information in completing a mathematical task, difficulty recalling information from long-term memory (attentional-sequential dyscalculia;' Badian, 1981). These students have difficulty keeping track of all of the information provided, frequently "lose their way" working through a task, forget information that they had learnt previously, generalizing or transferring ideas learnt, etc.

Rourke (Rourke & Strang, 1983) identified two types of arithmetic disability, characterized by information processing ability; a specific arithmetic disability group who had difficulty processing visuo-spatial and tactile-perceptual information and an arithmetic and reading disability group who had difficulty processing verbal information. Several investigations (for example, Share, Moffitt & Silva, 1988) have supported Rourke's model.

2) The learning environment assumes that the student can acquire and use spontaneously a range of general-purpose learning strategies. I noted earlier the assumption that students construct mathematics knowledge by using spontaneously a range of mathematics learning strategies.

The notion of the learning disabled student as a non-strategic learner (the "inactive learner hypothesis, Torgesen, 1980), less likely to activate spontaneously the range of strategies necessary for learning has been well-developed for students who have difficulty reading and writing (for a review, see Flood and Lapp, 1990). It has been less well-developed for mathematics. As I noted earlier two types of strategy can be identified; cognitive reasoning strategies and metacognitive or regulation and control strategies.

One of the difficulties associated with studying strategy use is the extent to which this use can be detected, monitored and measured. The technique of instructing pupils to "think aloud", that is to verbalize as they manipulate data, has not been shown to be immune from interfering with the learning situation. An alternative technique involves observing the effect of direct strategy instruction on subsequent performance.

Analyses of the errors made by mathematics underachievers support the inference that these students are less likely to use spontaneously a range of mathematics learning strategies; to initiate and implement a sequence of task-processing strategies, to read symbolic statements efficiently, to organize the ideas being learnt into semantile categories, and to retrieve relevant data from long-term memory. Teaching these students to use self-instruction strategies has led to improvement in working through arithmetic word problems (Montague & Bos 1986; Fleischner, Nusum & Marzola, 1987) in reading symbolic statements (Munro, 1989) and for retrieving information from long-term memory (Swanson & Rhine, 1985). These strategies are generally not developed directly in regular classes; most students learn them incidentally.

3) The learning environment frequently assumes that in the acquisition of a new idea, students can manipulate subordinate ideas in a relatively attention-free or automatic way, and can allocate most of their mental resources to 'building the new idea'. The importance of automatizing particular aspects of mathematics knowledge, so that these can be manipulated without the investment of mental attentional process for subsequent mathematics learning has been noted by several investigators (Garrett & Fleischman, 1983; Ackerman, Anhalt & Dykman, 1986). Evidence that mathematics underachievers have difficulty meeting this demand particularly for the manipulation of "basic number facts" has been provided by Fleischman, Garrett and Shepard (1982). As well of course, the issue of automaticity can be applied to the use of mathematics learning strategies, for example, the pupil activating the relevant processing strategies in an attention - free way.

4) The learning environment assumes that the pupils believe that they can learn mathematics and are motivated. Self-confidence in one's ability is an important aspect of mathematics learning; achievement and self-confidence are moderately correlated (Fennema and Sherman (1977) reported correlations of between .22 and .47). One's self-confidence determines in part how one will respond to success and failure, and the extent to which one is motivated to learn further. Dienon and Dweck (1978), categorizing students in terms of how they attributed success and failure, identified a "learned helpless" group. These students felt that success was beyond their control, failure was inevidable and that effort was useless because it would probably not lead to success. Kloosterman (1988) showed a relationship between attributional style, failure as an acceptable phase in learning mathematics and self-confidence; self-confidence was determined in part by what students told themselves about success and failure. In summary, students who lack self-confidence as mathematics learners and who attribute failure to their own lack of ability and who believe they can do nothing about it, are less likely to be self-motivated to learn mathematics, and less prepared to take risks, etc. The learning environment needs to take account of issues such as self confidence, the attribution of success and failure and self-motivation.

This section has examined the notion that mathematics curricula frequently make assumptions about how children learn, and while these assumptions are valid for most students, they can be shown to be less valid for underachievers. Further, these assumptions are not mutually exclusive. A mathematics curriculum that matches how these students go about learning will simultaneously take account of how they learn as well as providing them with the opportunity to learn how to learn more effectively.

The Mathematics Learning Centre as a Model for Intensive Mathematics Instruction.

The Mathematics Learning Centre (MLC) has been developed to meet the needs of three client groups simultaneously; underachieving students, their teachers and parents.

1) Mathematics underachievers: the aim here is to assist these pupils to gradually integrate themselves into their regular classroom learning situation, by helping them to:

- a) increase their mathematics knowledge,
- b) increase their repertoire and use of effective mathematics learning strategies, and
- c) improve their perceptions of mathematics, how it is learnt, and themselves as mathematics learners.

To this end the mathematics content on which the pupils work is the content that they are required to learn in their regular classes. The rationale here is to assist pupils to learn how to "carry one load more successfully" rather than expecting them to learn an alternative mathematics content. This frequently requires assisting the pupil to acquire as well preliminary or prerequisite knowledge. This approach, of course, necessitates a regular exchange between the pupil's classroom teacher, the Centre teacher and parents.

2) The parents of the underachieving pupils. It was recognized from the outset that the parents of the underachieving child are a key cornerstore of any successful program. Parents are frequently aware of their child's perception of mathematics and self-concept before the child's teacher, particularly if the child has shown negative attitudinal behaviours. In many cases it is the pupil's parents who refer the pupil to the Centre's program.

When the intensive teaching program begins, the pupils will usually be expected to complete regular homework tasks. Parents need to know how to support their children so that this work is optimally useful. Parents can provide a range of support services;

a) general non-directed support, for example, "It's OK if you can't finish it now. You can ask your teacher tomorrow".

b) showing that they have an awareness or understanding of their child's mathematics learning,

c) assistance with the present tasks at hand; the parents learn how to talk with their child about the current tasks and learn with their child, and

d) a broader educational role, encouraging their children to look for and discuss mathematical problems and situations arising in their everyday life.

Frequently, because of their emotional involvement with their child's past learning difficulty, parents frequently need to learn how to be supportive and how to assist in practical ways. To help them to learn how to do this, to see their child's learning difficulty in perspective, to see their child making progress and to learn what practical steps they can take, the program has at various times included several components:

a) prior to the child being involved in the program, parents complete an observational behavioural checklist that requires them to reflect on their child's mathematics learning history and to identify their child's strengths and difficulties;

b) during individual teaching sessions, parents observe for an increasingly longer time their child learning mathematics with the child's teacher discussing and explaining why particular steps were taken, and

c) group parent workshops in which the approach taken to mathematics learning is discussed in practical ways, and parents share problems and ways of dealing with them.

Throughout the program, teacher-parent communication and feedback is seen as important and a diary or journal is regularly maintained for this purpose.

3) Teachers interested in improving their ability to work with mathematics underachievers. Two related aims are important here; for the teachers to:

a) increase their repertoire of strategies for understanding and assisting mathematics underachievers; and

b) improve their perceptions of how mathematics is learnt and their self-concepts as mathematics teachers.

The teachers involved in the program have generally had classroom teaching experience and the in-service education component builds on their existing knowledge base. The in-service component is developed in two parallel formats:

a) an initial lecture-seminar-workshop; this is a "pre hands-on" activity in which teachers:

    i) evaluate their own experience and perceptions

    ii) examine how mathematics is learnt, why some children have difficulty, ways of assessing children's mathematics knowledge and strategies for facilitating learning in various areas, and

    iii) re-assess then knowledge, develop an in-service plan for themselves and prepare a goals statement.

b) the supervised hand-on component.

In this component the teacher works under supervision with the pupil over 10 to 15 sessions. This component involved:

a) developing an initial perception of the pupil's learning difficulty;

b) planning, implementing and evaluating the program, that begin in the Learning Centre and gradually moves to the pupil's school, and

c) overall evaluation of the teacher's change in knowledge and attitudes towards mathematical learning difficulties.

While the parts (1) and (2) above are superficially distinct (part (1) is assessment-diagnostic and (2) is teaching), this distinction generally becomes diffuse and blurred in practice, such that the teacher continually collects information re the pupil's mathematics learning, and modifies this perception as the program continues. An overview of the hands-on component is shown in Figure 1.

During the initial diagnostic planning stage the teacher develops an initial tentative plan of the most appropriate learning program by collating information from several sources:

a) the pupil's mathematics performance on reasonable tasks (a screening test, mathematics tasks given to the child in her/his regular classroom over the previous year) are error analysed in a series of clinical interviews. The aim here is to identify the conditions under which the pupil can complete on self-correct tasks, for example, when the pupil is instructed to verbalise the task, to visualize etc. The error analysis procedure is described in detail in Munro (1990).

b) the pupil's affective behaviours while working through mathematics tasks are also monitored, for example, behaviours characteristic of learned helplessness, extreme anxiety, impulsively, how the pupil attributes success and failure. As well, the pupil's attitudes towards mathematics, and the pupil's self-concept as a mathematics learner and monitored using a behavioural rating scale;

c) information re the pupil's overall mathematics learning in the regular classroom is also collated using a teacher information checklist that taps information in the following areas:

    i) the classroom teacher's perception of the pupil's mathematical difficulties;

    ii) the teacher's perception of the pupil's attitude towards mathematics, for example, does the pupil participate willingly in mathematics lessons, appears confused, all "at sea", anxious, depressed and withdrawn, more withdrawn in mathematics lessons;

    iii) the pupil's mathematics learning history; the nature of mathematics curriculum and teaching styles to which the pupil has been exposed;

    iv) classroom expectations; and

    v) the content currently under study.

d) information re the pupil's mathematics learning at home is monitored using a parent information checklist that taps:

    i) the parent's perceptions of mathematics;

    ii) the parent's perceptions of the pupil's attitudes towards mathematics at home;

    iii) how the pupil goes about learning at home; and

    iv) how the parents account for the pupil's mathematics learning difficulties.

e) general referral information, such as:

    i) possible causes, correlates of the pupil's learning difficulty;

ii) learning difficulties and strengths in other areas; and

iii) personal information re the pupil's family.

The teacher compiles an initial hypothesis about why the pupil hasn't benefited from past mathematics teaching and the conditions under which the child is more likely to learn from these data sources. This hypothesis is discussed with the pupil (for example, "During our discussion, when did you think you could learn easiest ? When we broke the problem up into small parts, when you said parts of it aloud, what you had to do, etc.?" It is useful for the pupil and teacher to reach a consensus here about how the pupil can learn best.

The pupil and teacher plan and negotiate an initial set of goals and program. The focus here is "What would you like to learn in mathematics?" It has been my experience that the majority of pupils involved in the MLC know what they would like to learn and generally suggest a set of realistic content objectives. When pupils are reluctant or have difficulty suggesting particular objectives, they can be given options such as "Would you like to learn more about... or ......?" From this discussion the teacher plans a teaching-learning program and discusses this with the group of Centre teachers, the pupil's classroom teacher and the pupil's parents.

This teaching-learning program is administered in 10 to 15 "teacher-pupil under supervision" sessions, as shown in Figure 1. Following each session teachers individually review and plan, and then present their ideas to the group of Centre teachers. Teachers are encouraged to support each other, to work work together to share resource and to monitor each other teaching. Each teacher is encouraged to update self-evaluations. During these sessions, communication with the pupil's classroom teachers and parents is maintained. The pupils are encouraged to monitor their own learning and progress.

At the conclusion of the program, the pupil and teacher each write overall evaluations and develop a plan for the pupil's continued learning.

What constitutes a successfully completed program for any pupil? A pupil is judged to have successfully completed a program when the pupil now feels that she/he can learn mathematics successfully; this is decided by the pupil and the centre teacher in consultation with the pupil's parent; measures taken into account are:

1) the pupil's level of achievement, particularly in relation to what is being covered in her/his regular class;

2) the pupil's self-concept as a mathematics learner; the pupil believes she/he can learn mathematics successfully;

3) the pupil's use of mathematics learning strategies, as rated by her/his Centre teacher;

4) the classroom teacher's rating of the child's performance in regular classes; and

5) the parent's perception of the child's approach to mathematics at home.

In the past, most pupils have finished the program when:

1) the pupil, Centre teacher, parent and class teacher agree;

2) 15 sessions have elapsed;

3) the child has achieved the specified goals (although some children frequently request to negotiate a second set of goals).

Evaluation of the Effectiveness of the Mathematics Learning Centre

The Mathematics Learning Centre project was (and is) an innovation in teacher inservice, pupil support and parent education, and, as such, ongoing evaluation of its effectiveness was a key component. The need for evaluation needed to be counter-balanced against the resources available for the evaluation. Given that there were virtually no resources for the project as a whole other than those that were generated by the project and that these were generally used for the purchase and development of learning and teaching materials, it is necessary to indicate at the outset that the evaluation component has never been conducted to my satisfaction. Instead, evaluation procedures were fitted in such a way that they provided partial answers to evaluation questions while making minimal demands on resources.

The evaluation questions asked were related directly to immediate purposes of the project:

1) How well are client goals (pupil, teacher and parent) being achieved at any time, and when does an individual client no longer need access to the services?

2) In what ways do the clients see the project being improved; areas of weakness, areas in which the delivery of services could be improved?

3) How well are gains made by clients (pupils, teachers) during the program sustained in the longer term?

The goals for each client group and the procedures used to assess goal attainment are shown in Table 1, taken from a 1988 report that I wrote (Munro, 1988). These criteria were used to monitor the gains made by 53 pupils who were involved in the Centre's program for 15 sessions in 1987. The students ranged in aged from 7 years to 13 years (median age ranged 9-10 years) are from grades 3 to 7. All were achieving at a mathematics level that was at least 2 years below their grade level. Each pupil's performance on each of the criteria was assessed at the commencement of the program and then repeated at session 11. This allowed a % gain score to be calculated for each student (% gain score = change in score / total number of items x 100), and a median score for the group.

Table 2: Gains made by Pupils Involved in 1987 Program.

Criterion	Assessment Procedure Used	Median % Gain
Achievement	1. Previous year's DMT	58%
	2. Proportion of set goals achieved by Session 11	74%
Use of Learning Strategies	Frequency of using each type	71%
Attitude Towards Mathematics	Frequency of positive-rated responses to attitude scale (42 item questionnaire).	83%
Performance in Child's Classroom	Frequency of positively rated behaviours (20 item behavioural rating scale).	68%
Parent Perception	Frequency of positively rated behaviours (using 24 item behavioural rating scales)	79%

These data show a substantial gain on all criteria.

To examine the extent to which the gains made by session 11 were sustained six months later, 49 of the 53 1987 pupils were examined on a number of criteria. The criteria and outcomes were as follows:

1) How valuable/effective/useful did the pupil judge the 1987 program to be? A 20 item questionnaire with a 3-point rating scale for each item was answered by each pupil. The average overall rating was 2.83 (between "good" and "very good").

2) Is the pupil still using the strategies learnt? The extent to which the pupil spontaneously uses "thinking aloud" strategies when working through current mathematics tasks was monitored. The % of pupils who used the strategies on at least 80% of possible instances was 71%.

3) How well was the pupil now able to achieve the goals that she/he set for her/himself during the program? The median % achievement was 83% (range 76% - 100%).

4) Has the pupil maintained a positive attitude towards mathematics? The median % of positive responses on the attitudinal questionnaire was 89% (range 82% - 90%).

5) How do the pupil's classroom teachers rate the pupil's current progress, for example, does the pupil continue to be engaged in mathematics lessons, takes learning risks spontaneously, has a positive attitude, attempts to solve problems without seeking assistance? The proportion of pupils rated as making at least adequate progress was 86%.

6) How does the pupil's parent rate the success or value of the program? Of the parent group, 93% rated the program as having been of substantial benefit for their child. They supported their ratings with a range of positive behavioural statements (for example, pupils attributing their current progress to their involvement in the program).

This evaluation supports the claim that the Mathematics Learning Centre program has been successful intervening in the mathematics learning of a group of pupils, who, at the beginning of the program, was seen as having severe mathematics learning difficulties. The gains in achievement, attitudes and mathematics learning strategies were shown to be sustained six months after the conclusion of the program.

The extent to which the teachers involved in the Centre's training program rated it was an effective teacher inservice program in 1987 was examined using a 24 item 5-point rating scale completed following the final teaching session. The mean rating score was 4.27, that is, a rating between "a valuable training experience" and "an extremely valuable training experience".

Conclusion

This paper was written in response to an invitation from the State Board of Education to describe the work of the Mathematics Learning Centre; and to show how the model on which it is based provides one option for the development of a more broadly-based intensive numeracy strategies program.

The Centre was and remains "homegrown", and differs in fundamental ways from the more traditional "clinic" approach (for example, as described by Englehardt (1985), Irvin and Lynch-Brown (1988) and Scheer and Henmger (1982). The more traditional approach was seen as inappropriate for several reasons, particularly because of the assumptions that this model makes about learning and teaching, and, to a lesser extent because the resources necessary for implementing such as approach were simply not available.

The aim was to develop a means by which several purposes could be achieved simultaneously within a structure that had a sound mathematics learning base and that was optimally flexible and versatile and could respond to changing needs, and that was "resource-lean" (in other words, that made maximum use of existing resources). The Centre, as it exists, is readily transportable. It provides teachers and schools with a perspective on how mathematics is learnt. It is my belief that this perspective can be readily incorporated within.

References

Ackerman, PT; Anhalt, JM & Dykman, RA. (1986) Arithmetic automatization failure in children with attention and reading disorders: Associations and sequela. Journal of Learning Disabilities, 19, 4, 222-232.

Australian Council for Educational Research. (1984) Progressive Achievement Texts in Mathematics. Hawthorn: ACER.

Badian, NA. (1981) Dyscalculia and m disorders of learning. In HR Myklebust, (Ed.) Progress in Learning Disabilities, Volume V. New York: Grune & Stratton.

Burton, L. (1984) Mathematical thinking: Struggle for meaning. Journal for Research in Mathematics Education, 15, 1, 35-49.
Shulman & Carey. (1984)

Cobb, P. (1986) Making mathematics: Children's learning and the constructivist tradition. Harvard Educational Review, 56, 3, 301-306.

Deer, AM. (1985) Conservation and mathematical achievement in the learning disabled child. Journal of Learning Disabilities, 18, 6, 333-336.

Englehardt, JM. (1985) Math clinic purposes: organisation or values. Focus on Learning Problems in Mathematics. Spring, 7, 2, 41-63.

Fleischner, JE; Garnett, K. & Shepard, MJ. (1982) Proficiency in arithmetic basic facts computation of learning disabled and non-disabled children. Focus on learning problems in mathematics, 4, 47-56.

Flood, J & Lapp, D. (1990) Reading comprehension instruction for at-risk students: Research-based practices that can make a difference. Journal of Reading, 33, 7, 490-496.

Garnett, K. & Fleischner, JE. (1983) Automatization and basic fact performance of normal and learning disabled children. Learning Disabilities Quarterly, 6, 223-230.

Hacian, H. (1962) Clinical symptomology in right and left hemisphere lesions. In V Mountcastle (Ed.) Interhemispheric relations and cerebral dominance. Baltimore: John Hopkins Press.

Hiebert, J. (1986) Conceptual and procedural knowledge: the case of mathematics. Hillsdale, NJ: Erlbaum.

Irvin, JL & Lynch-Brown, C. (1988) a national survey of US university reading clinics. Journal of Reading, February, 436-447.

Kamil, CK. (1985) Young children re-invent arithmetic. NY: Teachers College Press.

Kirby, JR & Becker, LD. (1988) Cognitive components of learning problems in arithmetic. Remedial and Special Education, 9, 5, 7-16.

Kloosterman, P. (1988) Self-confidence and motivation in mathematics. Journal of Educational Psychology, 80, 3, 345-351.

Kosc, L. (1986) Basic forms of dyscalculia. Focus on Learning Problems in Mathematics, 8, 3&4, 55-62.

Labinowicz, E. (1985) Learning from children. Menlo Park CA: Addison Wesley.

Leinhardt, G. (1989) Math lessons: a contrast of novice and expert competence. Journal for Research in Mathematics Education, 20, 1, 52-75.

Munro, JK. (1983) Self-esteem in mathematics learning: Empirical issues. Australian Journal of Remedial Education, 15, 2, 3-14.

Munro, JK. (1987) Mathematics anxiety: It's nature and identification. Success in Learning Mathematics, I, 2, 3-17.

Munro, J. (1989a) Error analysis in mathematics. Success in learning mathematics, 2, 6, 3-29.

Munro, J. (1989b) Reading in mathematics: A subset of reading. Australian Journal of Reading, 12, 2, 114-122.

Rourke, BP & Shang, JD. (1983) Subtypes of reading and arithmetic disabilities: a neuropyschological analysis. In M Rutter (Ed.) Developmental neuropsychiatry. New York: Giulford Press.

Scheer, JK & Henniger, MT. (1982) Math Clinic: an ideal setting for parental involvement. Arithmetic Teacher, October, 48-51.

Share, DL; Moffit, TE & Silva, PA. (1988) Factors associated with arithmetic-and-reading and specific reading disability. Journal of Learning Disabilities, 21, 5, 313-320.

Slesnick, T. (1982) Algorithmic skill is conceptual understanding. Educational studies in mathematics, 13, 143-154.

Steffe, LP. (1990) Mathematics curriculum design: A constructivist perspective. In LP Steffe & T Woods (Eds). Transforming children's mathematics education: International perspectives. Hillsdale, NJ: L Erlbaum Association.

Stiff, LV. (1989) Effects of teaching strategy, relevant knowledge and strategy strength on learning a contrived mathematical concept. Journal for Research in Mathematics Education, 20, 3, 227-241.

Torgesen, JK. (1980) Conceptual and educational implications of the use of efficient task strategies by learning disabled children. Journal of Learning Disabilities, 13, 364-371.

Van Erp, JWM & Heshusius, L. (1986) Action Psychology: Learning as the Interiorization of Actions in Early Instruction of Mathematically Disabled Learners. Journal of Learning Disabilities, 19, 5, 274-279.