## A language-based approach to middle-years mathematics[1]

**John Munro**

**What knowledge do we use to learn and manipulate maths ideas?** Work through some maths tasks and reflect on what you need to do to complete the tasks.

**Activity 1 :** A lesson in gimming

Look at these instances of gimming. What does gimming do?

4 gim 3 = 10     5 gim 3 = 13     6 gim 3 = 16

What does gimming do? Work out the value of

7 gim 3 =     8 gim 3 =     11 gim 3 =

Often when we learn an idea we need to modify our first impressions. What we thought was being done may not explain fully all parts of the idea. Here are more examples of gimming:

4 gim 2 = 7     5 gim 2 = 9

Work through the exercises below:

4 gim 7 =     5 gim 4 =     9 gim 8 =

Corrective feedback: 4 gim 7 = 22, 5 gim 4 = 17, 9 gim 8 = 65.

**Activity 2:** ***Place value*** Five cards.

You are given these cards. Your task is to make each of the numbers below. You can use some or all of these cards

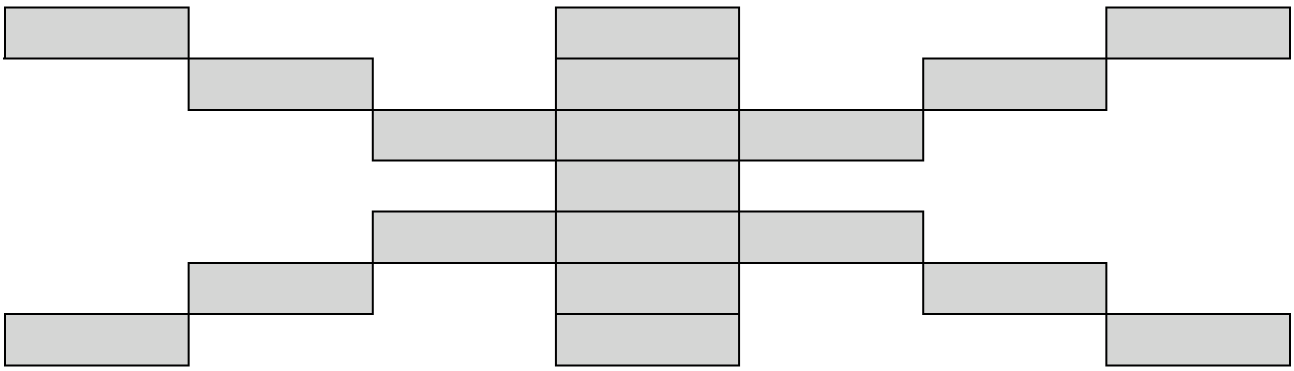
| | | | | |
|---|---|---|---|---|
| . | **3** | **4** | **5** | **6** |

| What is the smallest number you can make using the cards ? | What is the largest number you can make ? | What numbers between 4.3 and 6 can you make ? Say as many as possible. | Which number is nearest to 4.5 ? 4.4 4.56 4.6 | You need to explain how to decide which number is closest to a particular number ? How would you explain it. |
|---|---|---|---|---|

**Activity 3 :** ***Counting tasks.*** Suppose you have 19 tiles Write a number from 1 to 19 on each tile.

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 9 | 2 | 10 | 3 | 11 | 4 | 12 | 5 | 13 | 6 | 14 | 7 | 15 | 8 | 16 | 17 | 18 | 19 |

How can you arrange them in the following way so that the 3 arms add up to 76 ?



[1] A version of this paper was published in Munro, J. (2005). SUCCESS in learning mathematics. Third Editon. Melbourne : EdAssist

**Activity 4 :** Metroplana is the city of the future. Houses have maximum access to parkland. They are built on one side of each street only. Each street has more than one house. If there are 4 houses in a street, the numbers 1 to 4 are used but not in that order. If there are 5 houses in a street, the numbers won't be in order, that is, they won't go in the order 1, 2, 3, 4, 5.

- In Adam St no house numbers are in order. What is the smallest amount of houses that could be in the street ?
- In Beulah Street, no house numbers are in order. As well, no house numbers are different by 2, that is, you would not find house with number 5 beside the house with number 3 or the house with number 7. Again, what is the smallest amount of houses in Beulah St ?

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|---|---|---|---|
| **Activity 5** In each of the following, each letter stands for a digit. The same letter stands for the same digits whenever it appears. The same letter may stand for different digits in different tasks, Work out the value of the digit in each example. | GDG<br>+ GDA<br>AAKD | LPNE<br>IF ) FPEKP<br>FH<br>LE<br>EL<br>QK<br>HM<br>NP<br>NI<br>I | BACE<br>x 4<br>**CECDC** |

**Activity 7:** Mental arithmetic tasks

| | |
|---|---|
| Here is a strategy, Richard Feynman used to compute squares of numbers near 50.<br>1. Choose number to square, say 57.<br>2. Square 50 to get 2500.<br>3. Find difference between 50 and number you chose; 57-50 = 7.<br>4. Multiply difference by 100; 7 x 100 = 700.<br>5. Square the difference between original number (57) and 50; 57-50 = 7, so 7 x 7 = 49.<br>6. Sum your answers from steps 2, 4, and 5; add 2500+700+49 = 3249.<br><br>Does this always work for numbers near 50 ?<br>What about numbers less than 50 ?<br>What about numbers like 86 ? | Cori the camel lives at the edge of a large desert that is 1,000 miles across. Cori decides to take her banana harvest of 3,000 bananas to the faraway market that is 1,000 miles away on the other side of the desert. Cori can only carry a maximum of 1,000 bananas on her back at any one time, and she must eat one banana for every mile she travels. What is the greatest number of bananas that Cori can get to market? |

**Activity 8 :** A porcelain clock face falls from a wall and breaks into 3 pieces. The numbers on each piece at up to the same total. What are the numbers on the three pieces ?

**Areas of maths knowledge used to learn new maths ideas.** Across all of the content areas of maths, maths curricula usually require you to use what you know about

- Maths concepts, for example, what particular ideas mean, useful images linked with them, recognise examples of items, categorize maths tasks, recognise examples of a maths idea, transfer a maths idea to other contexts, recognise the numerator and denominator in examples of fractional quantities.

- Maths procedures or algorithms; you need to tell yourself the actions or steps in a procedure and recall how to apply action to unfamiliar tasks

- How to use and comprehend the language of maths, for example; how to talk, read, write, use maths vocabulary and symbolism of maths, for example, how to represent two fifths as a number, write in numbers a spoken number with places up to one hundred thousand., how to say each symbolic statement or number sentence in words, paraphrase and visualise s or word problems

- forming mathematical relationships; relating ideas, eg, ordinal, equality and inequality,

- automatize factual knowledge by talking about it; link individual ideas into larger chunks recall facts automatically, say the complete set of facts and individual facts.

- how to learn maths; the actions you use. There are two types of actions:

| | |
|---|---|
| ways of thinking about maths ideas, for example<br>• draw pictures of maths ideas, visualise it<br>• talk about ideas you are learning,<br>• ask *What does this remind me of?* This helps you link with what you know. | ways of managing their maths learning, for example, you<br>• plan your way through maths tasks.<br>• decide if solution is reasonable. |

- ***problem-solving strategies*** in maths; ways of thinking about mathematical ideas,

- ***their beliefs and attitudes*** about learning maths. You say how you feel about maths;

    - *what maths is like:* dull and boring, vs interesting, challenging, abstract, removed from the real world vs useful for solving real-life problems.

    - *how maths is learnt* - you must avoid making errors vs errors help you learn you mustn't guess or take risks vs risk-taking and guessing important in learning.

    - *themselves as maths learners*- I could never learn maths vs I find it hard but I can learn it.

**Key assumption of present approach**

Improved ways of thinking and talking about maths ideas assists understanding and comprehending → Improved maths outcomes

To learn any maths idea we can look at three phases: how well the teaching guides students to

- Orient what they know and to stimulate their relevant existing knowledge
- Learn the new ideas in a systematic 'learning friendly' way
- Consolidate and review what they have learnt.

You can use the maths knowledge framework we discussed to look at how well the teaching stimulates each aspect of knowledge at each phase of learning. What teaching procedures are used to stimulate /teach each aspect of knowledge at each phase of maths learning ?

| | Orient what students know | Learning the new maths ideas | Consolidate and review the new ideas |
|---|---|---|---|
| maths conceptual knowledge. | | | |
| maths procedures or algorithms | | | |
| how to use and comprehend the language of maths | | | |
| knowledge of mathematical relationships | | | |
| automatize factual knowledge. | | | |
| how to learn maths | | | |
| problem-solving strategies in maths | | | |
| beliefs and attitudes about learning maths | | | |
| integrate the various aspects. | | | |

***Teaching a numeracy perspective in our subject areas*** I gave an example of a recent series of lessons I developed with a Year 8 maths class on percentages.

I first met the class a few days I was to begin the teaching. I was introduced to the class and told them that I would be teaching them about percentages. I told them that I wanted them to collect as many examples and as much information as they could about percentages. They needed to be able to explain the information and examples they collected. They could bring newspaper cut-outs, brochures and any other relevant information to the first class.

As I was leaving, one of the students stopped me. Our dialogue is summarised as follows:

| | |
|---|---|
| Student | *We did percentages last year. I just want to tell you that I didn't like learning it and I don't know anything about them ?* |
| Me | (after a pause). *OK, that's alright. Suppose I asked you to eat 50 per cent of a cake. How much would you eat ?* |
| Student | (looking at me as if I were mad). *Half. Everyone knows that .* |
| Me | *Alright, if 50 % of the people in this room left, how many would be left ?* |
| Student | (again that look of 'where did he come from ?'). *Twelve.* |
| Me | *So you do know what 50 per cent is. What is twenty five per cent ?* |
| Student | (after a moment's thought) *A quarter.* |
| Me | *Well, so what other fractions do you know the per cents for ?* |
| Student | *I don't know fractions. I'm hopeless at them too.* |
| Me | *Write down some fractions* |

The student wrote down 1/2, 1/4, 3/4, 1/3, 1/5 1/8, 1/10.

| | |
|---|---|
| Me | *Which ones do you know the per cents for. Write down the per cents.* |

The student wrote down 1/2, 50 % 1/4 25 % 1/10, 10 %

| | |
|---|---|
| Me | *How do you know that 50 % is 1/2.* |
| Student | *It just is.* (hesitation). *No, wait, it's half a hundred. Twenty five per cent is a* quarter of 100. |
| Me | *So what would the per cent for 1/5 ?* |

This dialogue continued.

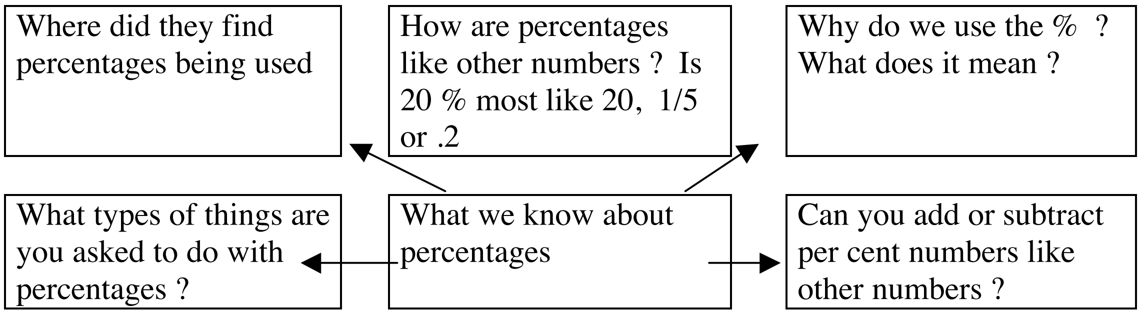
I have several reasons for including the discussion here:

- while the student believed she had no relevant existing knowledge re percentages, she in fact did. She didn't see this as 'mathematical knowledge'.

- when this knowledge was developed for specific episodes, that is, in a numeracy context, she was able to see what she did know and to extend it.

The activities in the first two lessons developed as follows:

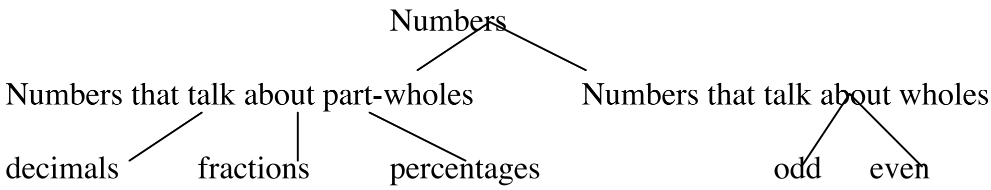
- We began the first maths lesson by having students in small groups pool what they knew / had learnt about percentages. We collated this knowledge by identifying first a number of questions we could ask about percentages. We draw these in the form of a concept map:




This activity was to allow students to clarify their existing knowledge about percentages.

- We looked at some of the particular episodes / instances of percentages collected by students . How were percentages used. We discussed the difference between what '34 people became ill' and ' 68 % of the party became ill' were saying. The groups invented other episodes of percentages and discussed what 'made a percentage episode'.

- We looked at how we could define the terms 'percent' and 'percentage'. What other words/ phrases could they use ? What type of thing is a percentage ? This led to us drawing the following network map to show how percentages were linked to other numbers:




- They discussed what they would ask if they had to interview a percentage number to work out its identity and the answers they might give. Each group had a percentage number about which it was interviewed. They made up difficult percentage problems for other groups.

- Each group was asked "What action are you doing when you are percenting ?" , needed to act out this and develop its own 'percenting action'. The class settled on a percenting machine being like a bread slicer that slices anything into one hundred equal sized pieces. They applied this action to several tasks, for example, to find a percentage of a quantity and to find the size of a whole when told what a percentage portion was equal to. The bread slicer became our icon for percentage problems.

- The students worked in pairs to apply the percentage procedures we had worked on to various numerical tasks and discussed how they did each type. They worked through mixed exercises and estimated each task first. They investigated how they could do percentages using their calculators.

- They again invented types of percentage problems in various real life contexts.

The focus in these lessons was to develop aspects of percentages within a maths lesson.

| Learn the mathematics ideas in **abstract symbolic ways** | Link feelings with the ideas | Learn cultural, social and historical aspects of the ideas |
|---|---|---|
| Can any fraction be converted to a per cent ?<br>Apply % procedures<br>Write your own procedures for working out percent problems ?<br>Prerequisite knowledge for dealing with numerical % problems ?<br>Categorising per cent tasksin different ways.<br>Deciding procedure to use ? | What feelings would you have if you were a number being per cented ? (you've been cut into 100 equal pieces)<br>What feelings help people to solve % tasks? (curiosity, etc) | When were %s first used in history ?<br>What cultures would be likely to use percentages ?<br>How will using calculators for working out percent affect how we use percentages ?<br>Will people in the future use percentages more or less ? |
| | Manipulating Percentages | |
| Brain-storm ideas----> concept map --> network map.<br>Discuss "What type of number is a % ? Is 20 % more like 20, 1/5 or 2 ?<br>Show on a network map<br>numbers<br>whole part of whole<br>fractions percents<br><br>Think aloud working through per cent tasks. Useful things to say ?<br><br>Teach a peer to solve % problems.<br>Other words to say 'per cent' ?<br>Where does the word come from ?<br>Make up 6 difficult % tasks<br>Debate :"Why bother to use % now ? They are unnecessary.<br>Paraphrase % tasks. | Collect, picture, draw, situations in which people use per cents in real-life situations such as 9% of population is unemployed, rate of paying interest on a loan.<br>What a percentage 'looks like'; 50 % of the group means 1/2 of the group, 25 % of it means divide by 4.<br>What do all episodes share ?<br>Invent icons of per centing. | What do you do to 'find a per cent' ? The per centing action ?<br>Groups of students acts out 'per centing' something .<br>Is there an opposite/ reverse action to per centing ? If 200 --->2,what action undoes this, that is, takes you from 2---> 200 ?<br>Make up models to show per centing.<br>What are other actions like per centing? What is special about the per cent action ? |
| Learn the new idea in language ways | Learn the new idea with quantities in real life contexts | Learn the new ideas in actions |

**A systematic approach to teaching any numeracy idea**. The following framework for teaching explicitly any idea in mathematics is taken from Munro (1991).

Key focus here : Teach students explicitly how to learn maths ideas by talking about them.

| Teaching focus : Challenge students to learn |
|---|

Teaching activities
- Introduce ideas as real-life problems to be solved, ask challenge questions.
- Present ideas that don't fit or seem to clash with what they know.
- Students guess and check; predict and compare with the actual outcome.
- Use novelty; make the novelty of new ideas stand out.
- Use open-ended tasks, students frame up questions and an action plan
- Use fantasy and imagination to lead students to questions and puzzles.
- students invent problems for peers, share and discuss solutions.

Teaching focus : Let students see where they will end up

Teaching activities : Have students imagine the learning outcome of the numeracy activity for example
- *This is what you will be know / be able to do when you have learnt ...?*.
- Visualise outcomes.
- Use of outcomes.

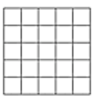
| Teaching focus : Stimulate what students already know |
|---|

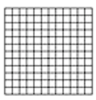
***Students differ in how they know a topic.*** Existing knowledge can be organised in abstract wayss, in images and in actions. Different forms in which existing knowledge can be stored:

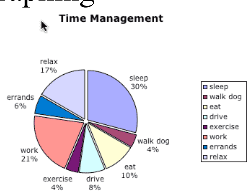
| abstract knowledge | imagery knowledge | procedural knowledge |
|---|---|---|
| 25 $2x + 5 = 11$<br>x4 | 4 packets of sweets with 25 in each | *this is what you do ..*<br>actions |

Teaching activities : Teach students to
- recall what they know about a topic in both experiences and abstract maths knowledge, talk about the maths terms and vocabulary, relationships and symbolism you assume they know. They describe what particular cues show, for example

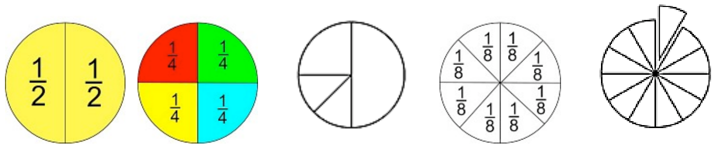
teaching unit on graphing





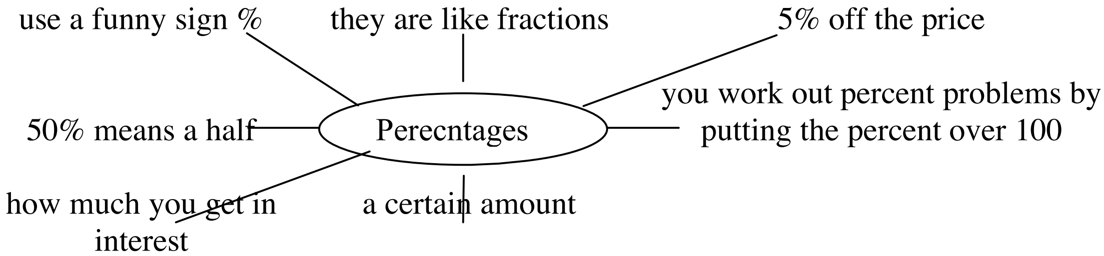


unit on equivalent fractions



- *What do you see in your mind/ think of when you hear the word "percentage" ? What can you draw / act out about the topic ? Brainstorm the topic in a think pair share activity.* Pupils say / write their knowledge of the topic in
  - free recall and concept mapping formats
  - a semantic web of possible ideas.

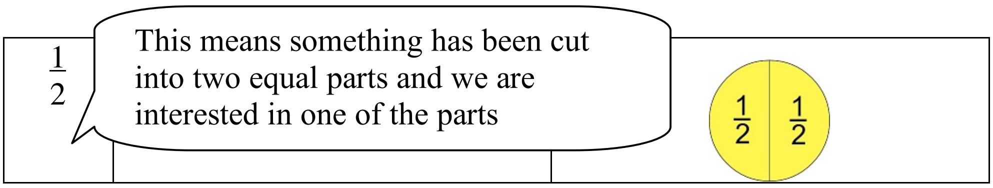
Collate what the groups know




If the teaching is mainly verbal, recode imagery and action knowledge to this form. Teaching activities to help students re-code what they know to the form of the teaching:

- *What can I say in 1 minute ?* Students tell their partners all they know about / what they think of /see in their mind/ think of doing when they hear a topic /challenge .
- *What does this picture remind you of ?*
- *What do these mean to you ?* Say 10 topic words to learners or groups and ask each to
  - act out or dramatise their interpretation of the words
  - visualise the topic
  - describe what the words remind them of
  - suggest questions that the words might cause them to ask.

Select the maths symbols and symbolic or number sentences you will assume the students can read, write and comprehend and chech they can do this, for example, for percentages




Help the students see that what they know is relevant and valuable

***Revise the factual knowledge they need to use automatically*** A week or two before you begin a topic, have students begin to revise the existing knowledge that they will need to use automatically when they are learning the new ideas. You can do this using
- class quizzes, partner quizzes
- maths card games
- mental arithmetic tasks on tape

| Teaching focus : Students learn new ideas first in' learner-friendly' ways |
|---|

Teaching activities : Introduce new ideas with quantities.

***Teach learn new ideas in familiar language with quantities first***. Teach students to

- do relevant physical actions to lead to mental actions and talk about them, draw and act on mental pictures of math ideas and talk about them, for example, do or visualise acting on specific quantities

> - ½ x 2/5 as repeating 2/5 'half a time'
> - .2 + 3 versus .2 + .3
> - converting 4 2/3 to an improper fraction
> - factorising 6a +8a
> - simplifying 2ab + 3a +ab
> - calculating a set of numbers {y} given a second set {x = 1, 2, 3, 4} and the link between them, eg, y= 2x + 1. $5x + 7 = 23$

*Say each task and visualise it. What picture does it tell me to make ? How will I act on the picture in my mind ?*

- Physical actions -> mental maths operations, internalise physical actions as mental actions

> say each of the following as an action
> - $\sqrt[3]{64}$
> - .2 + 3 versus .2 + .3, mentally add or subtract in steps of 1, .1,
> - 7 : 8
> - estimate and round, eg., 925 ÷ 34 ≈ 900 ÷ 30 = 30).
> - Solve $5x + 7 = 23$
> - $CSA = 2\pi r^2 + 2\pi rh$

*What action does it tell me to do ?*

- visualise symbolic descriptions of maths data, quantities number sentences, for example, visualise the following statements

  convert 4 2/3 to an improper fraction
  - Simpliify 2x + 2xy
  - Expand 2(3a + 4b)
  - $x^2 > 9$
  - $CSA = 2\pi r^2 + 2\pi rh$

*Say each statement and visualise. What picture does it tell me to make ?*

- Describe a quantitative or a numerical pattern, for example, talk about number patterns, for example
  - 2/3, 4/6, 6/9, 8/12 ….
  - .3, .5, .7, .9. 1.1, …
  - describe the pattern in .0023, .023, .23, .2.3

- talk about the maths ideas and relationships; for example- say a quantity in multiple ways

- 2/3 is the same as 4/6
- Other ways of saying ¾ are .75 and 75%, …
- say the equivalent for a mixed number, decimal or percentage, eg., 2¾ = 2.75= 275%
- .4 km = 400 m
- $(2)^5 = 32$. $2^0 = 1$

*How can I say this quantity in other ways ?*

- link the new idea with a known idea

| | |
|---|---|
| Get a maths task ready for an operation | • 2/3 x 2 1/5 = 2/3 x 1/5<br>• 1/3 + ½ = 2/6 + 3/6<br>• .27÷ 0.09 is the same as 27 ÷ 9<br>• 2 ¾ ÷ 1 ½ is the same as 11/4 x 2/3<br>• get $x^2 - 5x + 6 = 0$ ready to solve<br>• get √27 + √12 – 2√3 ready to simplify |

*How will I get the task ready to do ?*

- show the new maths idea first in familiar ways and then teach the conventional symbolism for reading and writing maths.
- talk about errors in a useful, realistic way.
- discuss new maths ideas in co-operative peer activity.

Teaching focus : Abstract ideas learnt

Teaching activities : Teach students to

- talk about the general pattern or procedure, paraphrase it, say how they will use it.

| | |
|---|---|
| Recognise order among numbers | say how they put the following items in order:<br>• 3/5, ½ 4/10, 1<br>• 10 % of 40, 50% of 8, 20% of 40<br>• 0.07, 0.6, 0.7, 0.23, 2.3<br>• -2, -5, +4, -1, +3 |

*How will I put these in order ? Which is the smallest one ? ….*

- read and write the new idea in mathematical symbolism

| | |
|---|---|
| Paraphrase maths data, read symbolic statements and number sentences | paraphrase<br>• A = lxw<br>• algebra such as 2(3a + 4b), 2x + 2xy<br>• directed numbers such as -2 + -5<br>• a formula such as $A = \pi r^2$<br>• x or a (say as "one group of a")<br>• |

*What does it say ? How can I say this in other ways ?*

- integrate or link the verbal description with concrete, pictorial, abstract, procedural aspects of the new idea.

How does the quantity change when you do the action

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|---|---|
| Say how a maths operation changes a quantity | • When you multiply a fraction by 2/2. 3/3, the value change<br>• How does simplifying change 9/12 or 2a +b – a +3b ?<br>• When you subtract -2 from -5 does it get more negative or more positive ? |

- organize their maths knowledge into categories, give a name to each category, talk about its characteristics and how they will recognise examples of the new idea.

| | |
|---|---|
| Categorize maths tasks, say when to use a particular maths operation | see table below |

| ready | not ready |
|---|---|
| 2/6 + 3/6 | 1/3 + 1/2 |
| 2/3 x 1/5 | 2/3 x 2 1/5 |
| 23÷ 3 | 23÷ .3 |
| convert 2/10 to decimal | convert 2/5 to decimal |
| simplify 5√3– 2√3 | simplify √27 + √12 |
| Solve (x-3)(x-2)=0 | Solve $x^2$ -5x +6 =0 |

*What does it say to do ? Is it ready to have the action done ?*

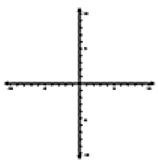
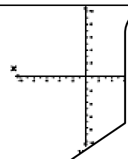
- Integrate the steps in a procedure,

| | |
|---|---|
| Do and co-ordinate 1,2, 3 computation steps mentally | Co-ordinate the steps in<br>• 2 1/3 +3 ½<br>• 6a +8a = 2(3a + 4b)<br>• 2 ¾ ÷ 1 ½ |

*What will I do first, second, ... ?*

- question the new maths ideas, say the questions they answer, make up difficult questions and tasks based on the ideas for peers.

| | |
|---|---|
| Ask questions related to the maths ideas being learnt | • can all numbers can be shown as a rectangular array?<br>• is getting a 6 on a die less likely than getting an odd ?<br>• does dividing always leads to a smaller number ? |

| | | |
|---|---|---|
| Students suggest questions that could be asked in a particular maths context and the questions that could be answered. |  |  |

*What questions could I ask about a function drawn here action done ?*

| Teaching focus : Students examine what helped them to learn |
|---|

Teaching activities : Teach students to

- 'think aloud' as they do tasks and to talk about their thinking. Ask : *What did you do to learn the new ideas?*
- use strategies explicitly, see how they help, record them and remind themselves to use the strategies in the future.

Model how you learn and have students share how they do tasks

| Teaching focus : Students respond emotionally to the ideas that they have learnt |
|---|

Teaching activities : Ask students to

- answer *"How do you feel about…? Did you find it interesting/boring?" What would make them more interesting?*
- say how useful the ideas are.
- say that their activity led to them learning the ideas and to value themselves as learners.
- give themselves positive feedback: *"Praise yourself for a job well done"*.
- comment on how they are feeling. Develop positive attitudes to learning.

- attribute success and failure functionally.
- take steps to reduce the likelihood of maths anxiety.

Teaching focus : Students see their progress

Teaching activities : *What have you learnt?* Say, write, draw, demonstrate.

- *What can you do now that you couldn't do earlier?* They tell each other.
- They tick off on their journey where they are now.

Teaching focus : Encode ideas in long term memory

Teaching activities : To store new knowledge in memory students

- prepare themselves to remember, 'tune themselves in': *I need to remember a new idea / procedure/ type of problem*
- say concisely what they want to remember and compress it into 3 or 4 major steps or parts: *"I need to say what I am going to remember as briefly as I can*
- *link the idea with what they know. What do the ideas remind me of? What are they like that I know? How are they different? "Where have I done things like this before?* Drawing concept map, categories charts, seeing familiar ideas in the new idea will help.
- use drawings, gestures or descriptive terms for idea*" What can I use to help me to remember this idea?*
- note use of idea, when they will use it in future *"What is the use of the idea? What does it do? Why it is useful? When will I use it? What type of problems it will help me solve?"*
- imagine themselves remembering idea at future time *"I imagine myself remembering the idea.*

To retrieve knowledge from memory allow gradual reconstruction;

- begin with a recognition task
- check that they know what to do to produce the items.
- ask them to produce the complete set of related items
- ask them to say the set of facts aloud.
- start to break up the complete set in the less demanding context.
- start to break up the complete set, in the more demanding context.
- break up the complete set further, in the less demanding context.
- break up the complete set further, in the more demanding context.
- recall individual facts, first in the less demanding context
- recall individual facts in the more demanding context Ask items out of order.

Teaching focus : Automatize aspects of ideas learnt

Teaching activities : Teach students to automatize their recall of mathematical ideas:

- recall the ideas often, say them and use them in a gradually wider range of contexts, for example, in card games that target recall, quizzes.
- speed up recalling the ideas and using them and anticipate when they may need to use them.

Teaching focus : Transfer the maths ideas to other situations, use them to solve problems, think creatively about them.

Teaching activities : Useful teaching procedures include having students

- speculate about the general rules and 'big pictures' suggested by the ideas they have learnt, investigate how do they fit with other maths ideas learnt.
- investigate practical uses of the ideas, for example, to what extent can the ideas be used to model phenomena in other areas of life.
- think creatively about the ideas by varying particular aspects of them and the assumptions made.
- examine paradoxes and puzzles based on the ideas.
- invent word problems that require the new ideas to be used in a range of situations, invent puzzles that use the ideas.

Teaching focus : Evaluate, assess learning outcomes

Teaching activities : Useful teaching procedures include having students

- discuss how they believe they will be expected to display the ideas in the future
- work in small groups to write assessment questions for peers, mock tests.
- practise recalling the ideas.

**How can you coach colleagues to improve their mathematics teaching ?**

To develop an implementation plan for coaching staff to in maths teaching:

To what extent can maths teaching be 'finely tuned' to include aspects of the learning recommendations described here ? You may need to guide staff to

- evaluate their current teaching practice in terms of the extent to which it targets each aspect of literacy; review how the ways of maths thinking are taught now,
- identify novel teaching procedures they can trial in their teaching,
- identify likely student outcomes to evaluate the effectiveness of the novel teaching procedures; develop monitoring /evaluation strategies,
- evaluate their fine tuned teaching and decide whether to add the novel teaching procedures to their teaching repertoire, to practise them and to broaden their use.

To assist staff to identify the aspects of learning it may be useful to include in their teaching, you can guide them to use the following questions to evaluate students' mathematics learning and identify where literacy and language strategies could be included :

| Do your coachees need to teach their students to | student behavioural indicators |
|---|---|
| • work their way through maths tasks systematically ? | |
| • build mathematics vocabulary ? | |
| • read symbolic statements efficiently ? | |
| • organize the ideas they learn into useful categories? | |
| • recall relevant data from long-term memory? | |
| • learn aspects of their knowledge in a relatively automatic "basic number facts" way? | |
| • learn self-confidence as maths learners? | |
| • improve their motivation to learn maths? | |
| • be more prepared to take risks when learning maths? | |
| • form more positive attitudes about their ability and belief they can learn it ? | |
| • show their knowledge in assessment tasks ? | |
| | |

To coach colleagues to include the language and literacy strategies in their mathematics teaching to improve student outcomes, they need a systematic way to teach each way of thinking while completing a maths task. Having decided the strategies to teach, staff can be guided to identify how the ways of thinking will be taught in topics they will cover in the next two terms.

| | Topic 1 | Topic 2 | Topic 3 |
|---|---|---|---|
| Paraphrase maths data, number sentences | | | |
| Visualise maths data, quantities, number sentences | | | |
| Say a quantity in multiple ways | | | |
| Physical actions -> mental maths operations | | | |
| Visualise acting on specific quantities | | | |
| Do 1,2, 3 computation steps mentally | | | |
| Categorize maths tasks | | | |
| Reflect on how they think through maths tasks | | | |
| Automatize numeracy knowledge, eg., table | | | |
| Use a systematic approach to a maths task | | | |

Having decided the strategies to be taught and the content to be used to teach them, staff can then develop a schedule for teaching each way of thinking. The following schedule is a learning –based sequence in which students are guided to learn to use each strategy gradually in a self managing and automatic way. Staff need to be coached to use teaching that allows this gradual independent mastery to be learnt.

| week | Students is told /shown | Students do /say | Students say and do | Practice + automatize | Link with other actions |
|---|---|---|---|---|---|
| 1-2 | | | | | |
| 3-4 | | | | | |
| 5-6 | | | | | |
| | | | | | |

Plan teacher learning activities for teaching each language and literacy strategy. Staff may need a range of professional learning/ teaching activities as part of the coaching to assist them to build each strategy into teaching. This helps to develop an implementation plan for building ways of thinking mathematically into topics they will teach.

| week | Leader models | Lesson /topic planning | Teacher trial in teaching, gather data | Teacher discusses, trials with colleague | Instructional leadership activity | Pooling group professional knowledge |
|---|---|---|---|---|---|---|
| 1-2 | | | | | | |
| 3-4 | | | | | | |
| 5-6 | | | | | | |
| | | | | | | |

The teaching may need to be modified to accommodate multiple ways of learning and individual differences in numeracy learning. Teaching strategies can assist teachers to cater for underachievement due to causes such as

- inadequate understanding of relevant quantities (decimals, percentages, algebraic ideas, etc.,)
- immature symbolic processing, understanding what maths symbolism says
- dysfluent, non-automatised use of mathematics ideas,
- poor sequencing of maths ideas,
- using inadequate metacognitive and self-management strategies and negative attitudes and dispositions to low self efficacy to their likelihood of success as maths students.

Procedures coaches can use to assess use of language and literacy in mathematical learning

- How well do the assessment procedures you currently use help you to assess each aspect of language and literacy in maths ? How can you use the approach here to review, evaluate and improve how you assess and screen maths ?
- What might be useful tasks to use to assess efficiently each aspect of language and literacy?

What does this approach mean for using a text book to teach maths ?