Reviewing teaching by examining what it does : Review from a learning perspective

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This paper is about teacher self-review, that is, how we as teachers can review what we do. There are, obviously, many ways in which we can do this. We are unconsciously doing this all the time, whenever we adjust to the learning characteristics of particular students and groups. We can also review how we teach by evaluating what we do when we are asked questions about our teaching style. A third way is to look at what our teaching allows students to do when they learn.

Reviewing for different purposes Each of these ways of reviewing how we teach has particular purposes or goals. Sometimes we need to modify how we work with a student or group. The student or group may be sufficiently different from what we are used to that our regular teaching procedures are not as effective. Sometimes we may decide we need to change how we will introduce or develop a particular topic. Our purpose may be personally motivated, for example, to increase our own level of excitement with what we are teaching or simply to audit our teaching.

Reviewing to improve learning opportunity One purpose for reviewing how we teach is to ensure that what we are doing leads to optimal learning. This is a long term' purpose; the outcome is likely to influence how we teach 'across the board'. It is more than responding to the needs of a student or group at one time, or responding to the need to teach a particular topic. It evaluates what we do and 'bench-mark' this against what has been 'shown to work' in effective learning and teaching.

A framework for reviewing maths teaching We teach maths with the goal of helping students change or add to what they know. The teaching is more effective when it both helps students to learn new maths ideas and to keep what they already know 'alive', to remember and apply what they have learnt. We need to put in place the teaching conditions that allow these to happen.

This is the approach taken in this paper. Its focuses on how students' knowledge changes and the conditions necessary for this. These conditions provide a framework for reviewing how we teach. The purpose of the review here is to examine the extent to which your teaching allows students to learn optimally. To review your teaching for this purpose you need to identify the aspects of learning your teaching fosters or supports. To look at this you need to look at what you actually do when you teach. You can then compare what you do with what are known to be conditions necessary for effective learning. The approach is shown schematically as follows:

Purpose of the review	To what extent does my teaching allow students to learn optimally
	What aspects of learning does my teaching foster or support ?
The data base for the review	What do I do when I teach ?

This paper identifies a number of these teaching conditions that lead to optimal maths learning.

To learn successfully, students need to 'interact with' the teaching information in an number of ways. They are more likely to learn when they

- frame up an explicit purpose or reason for learning an idea
- have an impression of the outcome or goal of the learning
- make links with what they already know
- change what they know in a particular contexts

- abstract and decontextualise the ideas they have learnt,
- respond with positive emotion to the ideas they have learnt,
- identify what they did to learn the ideas
- see themselves making progress with their learning
- store the new knowledge in long term memory
- practise recalling and applying it in a range of situations,
- automatise it and
- practise organising the knowledge for display in assessment contexts.

Effective teaching encourages learners to interact with the teaching information in these ways. The more of these interactions the teaching supports, the more effective it is. Each interaction can be developed in a variety of teaching contexts ranging from those more teacher-directed to the more open-ended enquiry problem-solving contexts.

This paper describes a procedure for reviewing your maths teaching by looking at the extent to which it supports these learning interactions. It does this by giving examples of teaching procedures that support or promote each interaction. It uses examples of content usually taught at CSF levels 3 and 4 to illustrate teaching procedures that are likely to foster each of these interactions. You can use these examples to review your teaching and to examine the extent to which it supports each interaction.

Challenge students to learn

Learning is more likely when students have a reason for learning, that is, when they have questions they want to answer or problems they want to solve. Effective teaching strategies that are more likely to challenge students to learn include

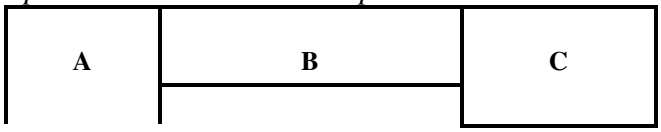
- introducing new ideas as real-life problems that students can solve first of all using any procedures they think are appropriate, for example, *You have four and a third pies . How many thirds are there altogether if you cut up the pies ?* These problems can come from everyday contexts with which the students are familiar, for example, from their environment, the areas in which their parents work, their communities.

- asking students to suggest questions the teaching might answer and what they think the answers are, for example, *We are going to learn how to find the area of a triangle. What questions can we ask about it ?* The questions that students suggest could include *How do you work out the area of a triangle ? Do triangles that are higher have more area ?*

- presenting ideas that don't fit or seem to clash with what students know. Examples are *In running trials Peter ran .56 km in 2 minutes and Ann ran .8 km in 3 minutes. Who ran faster ? By how much ? How will you decide ?*
  When you divide, do you always end up with a smaller number ? Divide 12 by one half. Does this fit with what you expected ?

- having students guess particular outcomes and then check them. This teaching highlights unexpected aspects of an idea and leads students to say *"That shouldn't have happened"* or *"these ideas don't fit together "*

  Which paddock covers the most space ? Which one has the longest fence around it ?



- using novelty. The teaching presents an idea in an unfamiliar context, for example, in a story or another narrative genre, a poster, a videotape, a demonstration, a model, a real-life context, to attract initial attention. Students learning about the Crusades in history may be introduced to challenges by asking questions such as *How far did the crusading knights travel from France to the Holy Lands ? How long would it have taken them ? How could we work this out ?* As they learn, they check their guesses against the teaching. They can use the questions to guide their learning and as it progresses, they can modify the question sequence.

- giving the class open-ended problems for which they frame up questions and action plans for solving them, for example, *Draw a poster for carpenters showing how to change between metres, centimetres and millimetres* or *Design a paper crane that will support a brick.* or *Write a tourist brochure for crusading knights travelling to Syria. What questions would you want to answer ?*

- using fantasy and imagination to develop ideas. *Prepare a travel brochure for interstellar travel telling them about distances, speed, time in light years.*

- having students invent similar problems for themselves and peers and encourages them to share and to discuss their solutions.

- using technology contexts to introduce potential problems to be solved. This context can be used both to present the problem and to to its relevance as a problem worth considering. Students' knowledge of caculators and computer games are rich sources of problems.

Review the extent to which your teaching encourages students to frame up challenges for themselves to learn ideas.

How often would you	Not at all	Some- times		Always
• introduce maths ideas as real-life problems that students solve ?	❒	❒	❒	❒
using any procedures they think are appropriate ?	❒	❒	❒	❒
• ask students to suggest questions the teaching might answer?	❒	❒	❒	❒
• present ideas that don't fit with what students know?	❒	❒	❒	❒
• have students guess particular outcomes and then check them ?	❒	❒	❒	❒
• use novelty ?	❒	❒	❒	❒
• use open-ended tasks in which the students frame up questions and an action plan for completing them ?	❒	❒	❒	❒
• use fantasy and imagination to develop ideas ?	❒	❒	❒	❒

As well as reviewing your teaching procedures, you can also review the learning behaviours that students display in learning mathematics. How often would your students

• pursue spontaneously their own interests in maths ?	❒	❒	❒	❒

- ask open-ended questions about maths ideas ? ❒ ❒ ❒ ❒ ❒

Develop an impression of the outcome of learning

Learning is more successful when learners have an impression of what the learning outcome will be like, that is, where 'their learning is going'. This impression helps them develop their pathway or action plan to achieve it and to plan and monitor the success of their learning activity. Typical teaching procedures that foster this interaction include

- helping them see what the outcome will be like, have a vision of the solution. In open-ended enquiry contexts they can describe what they think the solution will be like, what it will be able to do. In learning particular procedures, they can describe what they will be able to do having learnt. This impression of the outcome corresponds to the teacher's expectation of what is required. It is important that there be time during the teaching for pupils and teacher to explicate, clarify, and if necessay, negotiate this. Teachers' high expectations matter little if these do not map into student activity.

- encouraging learners to see where they are on their 'journey' through a maths topic, where what they are learning fits in and where they are going next. A programme of topics they will learn over the next month or term is a first step here.

- helping them see that their pathway is predictable from what they know now, showing how ideas they are learning extend ideas already learnt, where the current ideas 'fit' for example, *"You already know how to do add fractions that have the same denominator...Now you are going to learn how to do add fractions that have different denominators"* .

- negotiating periodically the next part of their 'journey through maths' with them.

Review the extent to which your teaching encourages students to see the direction of their learning.

How often would you	Not at all	Some- times		Always
• encourage learners to see where they are on their 'journey' through a maths topic and what they are learning next ?	❒	❒	❒	❒ ❒
• provide them with a programme of topics ?	❒	❒	❒	❒ ❒
• help them see that their learning pathway is predictable from what they know now ?	❒	❒	❒	❒ ❒
• negotiate the next part of students 'journey through maths ?	❒	❒	❒	❒ ❒

As well as reviewing your teaching procedures, you can also review the learning behaviours that students display in learning maths. To what extent would your students

- see the maths ideas they are learning located on a 'pathway' linked with ideas they had learnt earlier ? ❒ ❒ ❒ ❒ ❒
- be able to describe / draw parts of their 'journey through maths ? ❒ ❒ ❒ ❒ ❒

Make links with what students already know

Learning is more successful when students link the teaching information with what they know. There are several aspects to making these links.

Using maths knowledge stored in different formats. Students' maths knowledge is stored in different formats; in language, in imagery and in procedures. It is retained in real life episodes, in what they have learnt from playing computer games, watching TV and in playing sport, as well as under the topic 'maths' in their minds. Often students don't use their 'non-maths' maths knowledge because they don't believe it is relevant to 'school maths'. Effective teaching helps students link the teaching information with each of these forms. Teaching activities to help students do this include :

- asking students to draw / talk about what they already know about the ideas for example, *What do you see in your mind/ think of when you hear point three, point seven* ?

- asking students directly to link the new idea with what they know, for example, *You are learning how to work out 1/2 -1/3. What things like this have you learnt already ?*

- brainstorming a topic by students saying / writing all they think of about it. You can use co-operative sharing activities such as 'think, pair, share'. Students individually list the ideas they link with a topic such as 'decimals', pair with other students and share their thoughts.

- asking students to collect everyday information about topics they are learning, for example, *Collect newspaper articles and advertisements that refer to numbers greater than 10,000.*

- having students question the topic, for example *What questions can you ask about division?*

- having students recode their imagery and action knowledge of a maths topic to match it with teaching that is mainly verbal. As noted above, some of this maths knowledge is stored in real life experiences (for example, computer game experiences) that they don't believe is relevant to 'school maths'. Teaching procedures that help students re-code what they know to match it with the form of the teaching include having them talk about the images they make when they hear maths terms or tasks. Useful activities include
  - *What do I see in my mind when I hear point four ?*
  - *What can I say in 1 minute ?* To help, students can ask themselves*What do I know about.....? What other things is it like ?*

- having students listen to key words for the topic, such as *What do these mean to you ?* Say 8-10 topic words such as *multiples of three , twelve, divides into, times, multiply, write down, exactly, sixteen* and ask students in groups to
  - visualise the topic
  - describe what the words remind them of
  - suggest questions that the words might cause them to ask.

- providing positive feedback and correction for what students already know, letting them see what they already know about the topic and how it is valuable / useful for the learning.

All of these activities are important for the range of maths teaching contexts, from the open enquiry to the more procedure-directed aspects of maths teaching.

Using maths knowledge automatically Effective teaching has students revise the factual knowledge they need to use automatically. A week or two before you begin a topic, you can have students revise knowledge they will need to use automatically. Teaching procedures include

• class / partner quizzes. • maths card games • mental maths tasks on audio tape
These tasks can frequently be done independently, as homework tasks.

Getting knowledge ready to change Students can learn more effectively if they get their knowledge ready to change. In open-ended and teacher-centred learning contexts this involves them clarifying what they do and don't know about the problems or tasks at hand. They can:

- suggest questions they can ask about the topics, work in small groups to pool their questions and perhaps their suggested answers for example, :

  Type of number with a line and two 0s ?
  What are they ? Why are they written What other numbers are they like ?

  What questions can you ask about percentages ?

  How do you use them ? When / where do you see them used ?

- interview each another about the topic, for example, *Tell your partner all you know about .... Ask each other questions* for example, *when do you use percentages* ?

- say what they do and don't know about the topic, for example, *What do / don't I know about percentages ? What am I not sure of ?*

- work through prerequisite tasks, perhaps as homework activities.

- draw a 'map of the path' they have followed through the topic for example

Add fractions with same denominator	--->	Describe fraction in different ways; make equivalent fractions	--->	Improper fraction to mixed number
?		Add fractions with different denominators	<---	Lowest common denominator

What students believe both about the topic and their ability to learn it How well students learn a topic is affected by what they believe both about the topic and their ability to learn it . Have them 'bring to awareness' these beliefs. Have students

- respond emotionally to what they already know about the ideas, discuss what interests them and what is interesting to learn.
- say what they will do to learn the ideas.
- see that what they already know is useful and relevant to what they are learning.
- believe they can learn the new ideas and that we believe they can learn them as well.
- know that it is OK to ask questions about the maths ideas, take them apart, try things out, see how ideas fit.

Review the extent to which your teaching helps students make links with what they already know.

How often would you	Not at all	Some-times	Always
• ask students to draw / talk about what they already know about			

- the ideas to be learnt ? ❒ ❒ ❒ ❒ ❒
- remind students to link the new idea with what they know ? ❒ ❒ ❒ ❒ ❒
- ask students to say / write all they think of about a topic ❒ ❒ ❒ ❒ ❒
  by brainstorming it. ❒ ❒ ❒ ❒ ❒
- ask students to collect everyday information about topics they will learn ? ❒ ❒ ❒ ❒ ❒
- have students ask questions about the topic*?* ❒ ❒ ❒ ❒ ❒
- use co-operative sharing activities in which students share and ❒ ❒ ❒ ❒ ❒
  pool their thoughts about the topic to be learnt ? ❒ ❒ ❒ ❒ ❒
- have students recode their imagery and action knowledge of a maths topic into a verbal form ? ❒ ❒ ❒ ❒ ❒
- use activities in which students automatise prerequisite knowledge for a topic before learning it ? ❒ ❒ ❒ ❒ ❒
- have students suggest questions they can ask about the topic they will learn*?* ❒ ❒ ❒ ❒ ❒
- respond emotionally to what they already know about the topic they will learn ? ❒ ❒ ❒ ❒ ❒
- say what they will do to learn the ideas*?* ❒ ❒ ❒ ❒ ❒
- see that what they already know is useful and relevant to what they are learning *?* ❒ ❒ ❒ ❒ ❒

Learning in particular contexts

Students learn maths ideas by understanding them first in particular ways or contexts and then generalising or abstracting their understanding. Effective teaching allows them to

- explore the new ideas through actions in concrete situations or by using their episodic knowledge in open-ended ways in co-operative learning activities.
- guess the outcome of the learning using their existing knowledge for example, to estimate.
- use drawing and visualising to investigate the new ideas.
- develop mental actions to match the physical actions they have used to solve problems.
- manage and control part of the information input, pace their learning.
- receive corrective feedback that allows them to modify aspects of the new ideas.
- talk about their new maths knowledge in words, record it in familiar ways, for example, in drawings , words and familiar symbolism.
- see how their maths knowledge fits with everyday life, for example, links with teachnology, solving social problems.

Effective teaching procedures here include the following: the students

- act on either quantities or on pictures of the ideas and develop an action sequence for the ideas, for example, they learn to
  - rename mixed numbers such as 31/4 by acting on a concrete model of the amount, break each unit into quarters and count the total number of quarters.
  - read a circular measuring scale on which multiples of ten are labelled by using actions to segment the scale into smaller divisions.

  Some of the maths actions we want students to learn are procedures for
  - summarising and interpreting summary data,
  - manipulating spatial concepts and relationships
  - understanding chance and probability
  - using scaling, use ratio and proportion
  - using part-whole relationships, decimal, fraction and percentage concepts
  - using measurements, convert between different measurement scales in quantity, in space and in time
  - manipulating quantities in various ways
  - reasoning about numerical or quantitative patterns
  - using general formulae and statements to model or describe quantitative situations
  - comprehending area, perimeter and volume
  - thinking in two and three dimensions.

  Each of these can be learnt first as physical actions and then gradually internalised as mental actions.

- work on and solve problems in co-operative learning activities , share their maths ideas with peers, decide useful questions to ask about a topic, take turns to be the teacher and explain / justify an idea, act on each other's knowledge, write problems for the group.

- map episodes of maths ideas into abstract ideas and procedures. Suppose students are learning to convert percentages to fractions. In co-operative learning activities, such as 'think, pair, share' they pool episodes, analyse them and develop a more abstract understanding :

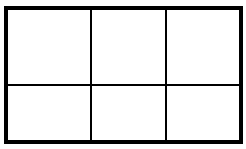
episodes:	--------->	abstract idea
50 % of the apples is half the apples 50 % of the people in the world are males	identify shared features decontextualize	50 % means half

- draw or visualise the new maths ideas, for example, visualise
  - how they would show the change in the average number of hours of TV viewing over the year by their class.
  - how they would act on a calculator so that they change the number 2.067 to
    - 2067
    - 2007

- show in various ways what they know as they learn the new ideas
  - first in familiar ways (words, drawings, familiar symbols), for example, draw .24 + .3 = using ▌ for ".1" and "-" for .01 and represent the task as ▌▌ ---- ▌▌▌ and the total as ▌▌▌ ▌▌ ---- or .54

  - later learn the conventional symbolism for reading and writing maths statements .

  The teaching can have them
  - translate their drawings of maths ideas into numbers using verbalisation as a bridge;



A third of a half is a sixth

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

- match number statements with drawings of maths ideas.
- write spoken number statements for ideas they are learning, interpret them in words or using concrete materials, visualise them, match them with everyday situations.
- describe how a number form is like and different from forms learnt earlier, for example, writing addition of fractions is like writing number addition equations.
- use maths games and related activities, develop their own games.

- talk about their new maths knowledge, for example,
  - they say *Three quarters is the same as six eighth* ; *you need twice as many eighths because two eighths match each quarter* to consolidate an idea they have learnt.
  - *Two wholes and three tenths add four wholes* for $2.3 + 4 =$
  - say $A= \pi r^2$ as *to find the area of a circle you square the radius and multiply this by pi.*
  - to order the set 6.07, 6.004, 6.2, 6.14, they say each number in tenths, hundredths, etc and compare them.

- practise applying the procedures they are learning, initially with sufficient modelling and support. Students can do this practise both individually and in small groups. During the early stages of any application they need to be encouraged to say what they will do and what each task says. Some may need to use concrete materials and/or pictures to support this learning. As their understanding of the procedure develops, the models and support can be gradually phased out. Once they have achieved a level of competence in the procedure, students may be able to practise it away from the classroom.

- manage and control aspects of learning at any time, for example, how much information they need to handle, pace their learning, see what they have learnt, that it is acceptable to change their mind during learning and to take risks.

- receive and make use of correction and corrective feedback. Effective teaching will
  - ensure that errors are corrected; *You are on the way / You're almost right*..
  - model how to to use correction and errors to sharpen their understanding; making an error means having another go. It avoids the 'one-off' one bite of cherry mentality.
  - shows that negative consequences do not necessarily follow from making an error.
  - points out that errors don't mean that students will do badly on a future test.
  - encourages students to tell themselves
    - *Everyone makes errors learning mathematics*
    - *Although I made errors, I can still learnt it. It didn't matter how many errors I make along the way.*
    - *It doesn't matter if I make an error. I can have more goes at it.*

Review the extent to which your teaching helps students change what they know.

How often would you have students	Not at all	Some-times		Always
• represent the ideas they are learning as an action sequence ?	❒	❒	❒	❒
• visualise the new maths ideas by teaching them to use icons				

and then to form mental pictures for the ideas ?	❒	❒	❒	❒	❒
• learn mental actions to match the physical actions ?	❒	❒	❒	❒	❒
• talk about their changing maths knowledge ?	❒	❒	❒	❒	❒
• say number sentences and tasks and analyse what they say ?	❒	❒	❒	❒	❒
• recode episodes of maths ideas into networks and procedures and vice versa ?	❒	❒	❒	❒	❒
• show what they know first in familiar ways (in words, drawings and familiar symbols) ?	❒	❒	❒	❒	❒
• use co-operative learning activities for students to learn new maths ideas through exploration and investigation ?	❒	❒	❒	❒	❒
• learn positive attitudes to the ideas they are learning ?	❒	❒	❒	❒	❒

Abstract, generalise, clarify the ideas learnt.

The second phase of learning maths ideas involves students understanding them in more general, abstract ways. The teaching here encourages them to

- integrate explicitly the contextual, abstract, procedural and emotional aspects of an idea
- put their maths knowledge into categories.
- link the ideas more broadly with what they know.
- question the new maths ideas
- manipulate the conventional symbolism describing the new ideas.

This phase is important for both the more open-ended enquiry and the more teacher directed approaches.

Teaching procedures that can help them learn maths ideas in these ways depend in part on the ideas to be learnt. Effective teaching procedures will:

- have students integrate the abstract, contextual, procedural and affective aspects of an idea. They
  - summarise the ideas, extract the main idea from specific tasks, develop a procedure or formula.
  - discuss contextual aspects, real world situations in which they might use the ideas. *In what contexts do you need to find the area of a rectangle ? What images / icons can I use to remind me of it ?*
  - do the actions or procedures that go with the ideas: *What actions go with the idea?*
- have students look at the new ideas from various angles, for example,
  - the positive and negative aspects of the ideas; *What are good / bad things about using calculators a lot to do maths ?*
  - how the ideas might be used in the future; *What new things might calculators be able to do in the future that they can't do now ?*
  - how the ideas have changed what we know, how we do things. *What things might we not be able to do if we didn't have a formula for finding the area of rectangles ? What maths procedures would be hard to do without calculators ? How did people do these before calculators were invented ?*

- have students put their maths knowledge into categories. Students can learn to classify

word problems into instances of | addition | subtraction | multiplication | division |

number facts into categories such as | addition facts to10 | ties | adding 9, 8,7 |

perimeter and area tasks | distance around the edge
perimeter of a rectangle
circumference of a circle | area
finding the space inside |

Effective teaching procedures will have students

- categorise physically instances of each type and gradually categorize the tasks 'in their heads'. Give the class a set of 'mixed tasks', with tasks that require different procedures for solution. Students need to look at each task, decide what type it is and say how they will solve it. You can use this format for solving word problems, too.
- decide how instances of each type are alike and differ.
- describe what they will look for when classifying instances of each type.
- produce their own examples of each type.

- have students question the maths ideas they are learning. They can
  - suggest the questions the ideas answer, for example, when shown a worked task, they suggest what the task might have asked them to find.
  - answer questions that direct them to infer; to reason about ideas, look for patterns, for example,*Why did you ...?* for example, "*Why did you rename 43 as 30 add 15 ?*"
  - answer questions that ask them to generalise, for example, *Do you always* (get a smaller number when you divide)?" and to analyse their understanding, for example, "*How does .. fit in* ?"
  - work on open-ended, questions that direct them to explore alternatives. *"What might we do here ?" "What do you think will happen ? "* You may follow up these questions with "*Why do you think that ?* " *"But what about .. ? Do you mean.. ?"*

- provide correction and corrective feedback that helps them modify their more abstract understanding of ideas, for example, how well they decide when to use particular procedures.

Review the extent to which your teaching helps students abstract and generalise the ideas they have learnt.

How often would you have students	Not at all	Some-times	Always	
• represent the ideas they are learning as an action sequence ?	❒	❒	❒	❒
• integrate explicitly the contextual, abstract, procedural and emotional aspects of ideas learnt ?	❒	❒	❒	❒
• look at the new ideas from various angles	❒	❒	❒	❒
• put their maths knowledge into categories ?	❒	❒	❒	❒
• link the ideas more broadly with what they know ?	❒	❒	❒	❒
• question the new maths ideas ?	❒	❒	❒	❒

- manipulate the conventional symbolism describing the new ideas? ❒ ❒ ❒ ❒ ❒

Students examine what helped them to learn

Maths teaching is more effective when it helps students learn how to learn maths, to

- think about maths ideas in different ways, for example, visualise, talk about or act them out.
- learn how to 'think mathematically'
- learn how to work effectively through tasks, to plan their way through tasks
- take effective action when ideas are difficult to learn
- become aware of what they do when they learn maths, to keep track of what they do, to talk about how they learn maths.

Teaching procedures that foster this learning include the following

- students describe how they learnt new maths ideas and discuss whether visualising or verbalising helped. *What did you do to learn the new ideas ? Did making a picture help ? What new things have you learnt about how to learn ?*

- teach strategies in an explicit way and have students try them out, see if they help them learn maths and if so, keep a record that will remind them to use the strategies in the future. They can do this on cards, for example, to work systematically through a task or problem they can learn to direct their attention to specific aspects at a time:

What does it tell me ?
Will reading it aloud help ?
Will drawing a picture help ?
What do I have to find ?
What type of problem is it ?
What does it remind me of ?
How is it like problems I have done ?
What procedures will I use ?
What does the first / second / third part say ?
Does the answer seem right ?
Have I answered the question asked ?

They can practise applying these to unfamiliar or difficult tasks 'in their heads'.

- students 'think aloud' as they do tasks and teach the language for talking about thoughts.
- students model for each other how they learn and trial each other's actions.

Review the extent to which your teaching helps students learn how to learn maths and to keep track of what works for them. How often would you

	Not at all	Some-times		Always
• remind students to think about maths ideas they are learning in particular ways, for example, to visualise them or imagine them as actions ?	❒	❒	❒	❒ ❒
• teach your students how to talk about the thinking they use while they are doing maths tasks ?	❒	❒	❒	❒ ❒
• have students think aloud as they work through maths tasks ?	❒	❒	❒	❒ ❒

- have your class collate and add to a set of 'ground rules for learning maths? ❒ ❒ ❒ ❒ ❒
- have students assemble and add to their personal lists of useful maths learning strategies ? ❒ ❒ ❒ ❒ ❒
- have students share their maths learning strategies and take turns to try out other students' strategies ? ❒ ❒ ❒ ❒ ❒
- teach your students how to plan their way through tasks ? ❒ ❒ ❒ ❒ ❒
- teach your students what actions they can take when they find maths ideas difficult to learn ? ❒ ❒ ❒ ❒ ❒
- become aware of what they do when they learn maths ? ❒ ❒ ❒ ❒ ❒
- have your students keep track of what strategies helped them while they were learning maths ideas ? ❒ ❒ ❒ ❒ ❒
- encourage students to see their way through tasks or problems when beginning the task or problem ? ❒ ❒ ❒ ❒ ❒
- encourage students to see possible 'danger areas, dead-ends coming up in tasks they are doing ? ❒ ❒ ❒ ❒ ❒

Students see themselves making progress in learning

Students learn most effectively when they can see themselves making progress. Teaching procedures that will help them do this include

- asking students at the end of each maths lesson to review and say what they have learnt. This can be done in a think-pair-share activity. Students identify what they have learnt, compare this with the outcomes of a partner and then share their outcomes another pair:
  What have you learnt ? Say, write, draw, demonstrate.
  What can you do now that you couldn't do earlier ?

- Students record in a diary or journal the maths ideas that they have learnt, for example

  14/7 Today I learnt to do word problems about going on a picnic.

  16/7 We worked on take-aways. Some of them were:

862	916	*I have to remember to say each*
-395	-253	*column to see if it is ready to take away.*

  17/7 We did a table quiz. I got seven right.

- students tick off on their journey through the maths program where they are now.

- students use devices such as the following format to show what they believe they know well and the ideas they are still allowing themselves to learn. New tasks are written on cards. Learners gradually move cards across as their proficiency improves.

New table	Not sure	More sure	Really sure	Know table perfectly
				3 x table

7 x table | | 9 x table | |

Review the extent to which your teaching helps students see themselves making progress in learning. How often would you have students

	Not at all	Some-times		Always
• at the end of a maths lesson review and say what they have learnt ?	❒	❒	❒	❒ ❒
• record in a diary or journal the maths ideas they have learnt ?	❒	❒	❒	❒ ❒
• tick off where they are nowon their journey through maths ?	❒	❒	❒	❒ ❒
• show the aspects of a topic they believe they know well, the aspects they are learning and the aspects they have yet to learn ?	❒	❒	❒	❒ ❒

Students respond emotionally to the ideas that they have learnt

Students are more motivated to learn in the future if they link positive feelings or emotions with the ideas when they learn them now. When their knowledge of the topic is retrieved at a later time, you want it to tell them that

- the topic is useful and / or interesting to learn
- they can successfully learn more about the topic
- learning the ideas will help them along their journey.

Useful teaching strategies include having students

- respond emotionally to the ideas they are learning. They answer the questions *"How do you feel about .. ?" "Did you find this interesting/boring/ to learn ?"*

- see the usefulness of the ideas, comment on how interesting and / or useful they found the ideas. *What would make them more interesting ?*

- value themselves as learners, that it was what they did that led to them learning.

- give themselves positive feedback such as "*Praise yourself for a job well done*" and evaluate how well strategies worked for them; *"I tried hard, used the strategy and did well"*.

- comment on how they are feeling, that it is acceptable to say *"This isn't bad; I feel I am making progress"*, that they are doing things they couldn't have done earlier.

- develop positive attitudes to learning; value curiosity and a preparedness to enquire.

- reflect on various aspects of mathematics; they can record in a journal their maths experiences, their feelings about mathematics they are learning and questions they might like to ask about the topics they are learning.

- see themselves as masters of numbers, making the maths work for them. To achieve this

    - encourage students to use maths procedures to solve problems that arise for example

"I want to get to the football about an hour before the game starts. It takes the tram 35 minutes to get from my stop to the stop near the ground. I live about 10 minutes from my stop. What is the latest time that I should leave home by?"
"What are the chances of winning Tatts lotto?"

- apply maths to themes or interests , for example, a hobby or a trip. Newspapers are a useful context. Maths can be used to solve problems, to model relationships, for example numbers used in the recent Voyager 2 trip, in various occupations.
- use games and related activities, particularly ones that students make or modify.
- the history of maths, famous mathematicians, their contributions, the conditions under which they worked.
- encourage estimation in real-life situations, for example,
  "How many steps would you take to get to Fred's house ?",
  "How long would to take you to ride your bike to the oval ?
  "How much milk do you drink each week ?"
  Students estimate, check and discuss how estimates could have been closer.

Review the extent to which your teaching helps students respond emotionally to the ideas that they have learnt. How often would you have students

	Not at all	Some-times		Always
• respond emotionally to the ideas they learn, discuss their feelings about them, how interesting they are, making them more interesting?	❒	❒	❒	❒
• see the usefulness or value of the ideas ?	❒	❒	❒	❒
• value themselves as learners, see that it was their activity, what they did, that led them to learn ?	❒	❒	❒	❒
• comment on how they are feeling while learning maths ?	❒	❒	❒	❒
• see that curiosity and a positive attitudes to enquiry is valued ?	❒	❒	❒	❒
• attribute their success functionally and see a difficulty learning an idea as temporary, something that can be 'turned around' ?	❒	❒	❒	❒
• see themselves as masters of numbers, controlling the maths ideas and can make them work for them ?	❒	❒	❒	❒

Encode ideas in long term memory

Effective maths teaching helps students remember what they have learnt. To do this we need to teach students to store and remember maths knowledge; we need to help students

- store new knowledge in memory by linking it with what they already know and
- retrieve or recall it by gradually reconstructing it.

Our teaching need to work on both parts.

Storing information in memory To help students store new maths ideas in long-term memory, effective teaching has students

- say what they have learnt and what they will remember. In pairs they can tell each other what they will remember. As well, they can draw a picture to remind themselves of the idea. If there are several ideas, they can compress them into 3 or 4 major steps or parts.

- say what do the ideas remind them of, how they are similar to different from what they already know. You can teach them to ask themselves *What do the ideas remind me of ? What are they like that I already know ? How are they different ?*
- think of things that can help them remember this idea, for example, a drawing, or a gesture.

Retrieving information from memory by reconstructing it Effective teaching helps students retrieve the knowledge from memory, by rebuilding it gradually, using the following sequence:

- begin with a recognition task to check they can recognise what they have learnt ; "*We were working on a times table yesterday . Was it the four times, three times, or ten times table ?"*
- check that they know what to do to produce the items, for example, can they remember how to count in nines to produce the nine times table or use MAB to subtract by renaming.
- ask them to recall the idea in the least mentally demanding situation. For the subtraction algorithm this may involve manipulating MAB materials or pointing to the digits and for the nine times table in may involve writing the whole table in order.
- ask them to recall the idea in the more mentally demanding situation. For the subtraction algorithm this may involve saying what was done first or second. For the nine times table it involves saying the set of facts aloud.

This reconstruction process usually take less than 10 minutes. You can repeat it in an abbreviated form, at the start of the next 2 or 3 sessions, with the students doing more of the work each time.

Review the extent to which your teaching helps students recall the ideas that they have learnt.

How often would you have students	Not at all	Some-times		Always
• say what they have learnt and intend to remember ?	❒	❒	❒	❒
❒ • draw pictures to remind themselves of ideas they have learnt ?	❒	❒	❒	❒
❒ • say what do the ideas remind them of ?	❒	❒	❒	❒
❒ • think of things that can help them remember the idea ?	❒	❒	❒	❒
❒ • practise retrieving information from memory by rebuilding it ?	❒	❒	❒	❒
❒ • ask them to recall the idea in the least mentally demanding situation initially ? ❒	❒	❒	❒	❒

Automatize aspects of ideas learnt

When students learn any maths idea, they need to use what they already know. The teaching usually assumes that students can use parts of their knowledge automatically. You would be aware of many examples of this.

When we want students to learn the subtraction algorithms, for example, we expect them to be able to recall the subtraction from 20 number facts relatively automatically. If these can recall these automatically, they can give most of their attention to the new idea they are learning. If they can't, they often have difficulty seeing the new idea among the other maths procedures they are needing to handle. The more information we recall automatically, the more attention we can give to new ideas. Areas of maths knowledge usually assumed to be automatized at the middle - late primary level include

- addition and subtraction tasks to 20, tasks involving tens and hundreds
- multiplication table tasks and division tasks based on the tables,
- counting in increments of 2, 3, 4, ... 10,
- place value to 100 and then to 1,000,
- fraction tasks involving small numbers and the number of parts that make one whole and
- decimal tasks involving inter-converting the simplest fractions, percentages and decimals.

To help students automatize their recall of mathematical ideas, the teaching needs to help them

- learn the ideas first in an attention demanding way; they show they can learn the ideas.

- recall the ideas often, say them and use them in a gradually wider range of contexts.

- speed up recalling the ideas and using them and anticipate when they may need to use them, for example, in games in which they match equivalent information such as matching '20 - 12=' and '8', in rapid quizzes.

Review the extent to which your teaching helps students automatise the ideas that they have learnt.

How often would you have students	Not at all	Some-times		Always
• revise knowledge they have learnt earlier in preparation for content they are learning currently ?	❒	❒	❒	❒ ❒
• speed up recalling the ideas and using them ?	❒	❒	❒	❒ ❒

Where to from here ?

This paper has led you to review your maths teaching in terms of extent to which it fosters essential types of activities learners need to use. You can use this to

- identify areas of your teaching that y ou might decide to modify over a period of time.
- develop an action plan for making these changes, clarify what aspects you might work on at any time and how you will monitor your progress.

There are, as you know, many other dimensions of teaching not covered directly in the above discussion. Some of these can, however, be examined indirectly from this analysis:

- individual differences in learning can be taken into account in terms of the set of interactions. Students differ in how they perform each interaction. The teaching procedures are intended to take account of these. These can be used to develop and trial individually tailored teaching programmes that can then be evaluated as part of a teacher's on-going self-review.

- classroom management and discipline procedures; because the set of interactions is intended to maximise active student involvement in the maths learning, minor disciplinary events due to disengagement, students not linking what they know with the teaching, students being bored and not seeing themselves maiking progress, are minimised by implementing the types of teaching procedures described.

- assessment, monitoring learning progress in a formative way and providing feedback to students and teachers are essential components of the set of interactions. Students monitor

their progress regularly. Teachers can observe how students implement each of the interactions and note those for which some students need additional assistance.

To review your effectiveness in these areas, you can examine the appropriate sections of the interactions and note the teaching procedures used.