Diagnosing learning difficulties in maths learning

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To diagnose maths difficulties, look at
- what evidence is there of a maths difficulty ?
- what maths the student knows.
- how can the maths disabilities be explained ?

Describing maths performance

To collect the samples of maths for later analysis, keep a record of
- the tasks, the conditions, the learner's output,
- the time taken.
- what learners say as they work through tasks.
- note learners' level of stress and any behavioural indicators of this.
- how maths ability changes when learners are cued to operate in particular ways.

Which test to use ?

1. Diagnostic Mathematical Tasks : Survey Tests for Junior Grades Prep to 6 (Schleiger, 1980; Gough, 1990) . Each test covers skills grouped into two parts:

Part A: Pure Number; numeration, place value, counting, recalling number facts, the four operations, number sentences and fractions.
Part B: Applied Number; length, time, money, space, capacity, mass, statistics and graphs.

2. KeyMath Revised (Connolly, 1988) comprises scales in three areas
- Basic Concepts; Numeration, Rational Numbers and Geometry
- Operations; Addition, Subtraction, Multiplication, Division and Mental Computations
- Applications; Measurement, Time & Money, Estimation, Interpreting Data, Problem Solving.

Each subtest covers 3 or 4 'domains'.
Limitations of KeyMath
- the usefulness of the norms for Australian conditions.
- the language used by many of the items is comparatively complex.

3. Progressive Achievement Tests in Mathematics measures knowledge for grade levels 3-5, 5-8 and 6-9 (Levels A, B and C respectively) in the areas of
- number
- computations
- fractions
- statistics +graphs
- measurement + money
- spatial relations
- relations + functions
- logic and sets.

4. ACER Tests of Basic Skills : Aspects of Numeracy (McQueen & Doig, 1997) sample number, space and measurement at the upper primary levels at three levels, A, B and C.
Number is assessed by having students
- apply arithmetic operations to whole numbers
- manipulate counting patterns
- arrange numbers in order
- manipulate place value
- select the appropriate operation to solve word problems
- read number sentences
- read decimal numbers
- add and subtract decimals, add two fractions
- sequence decimals and fractions

Teacher made mathematics tasks Examples of continua of maths knowledge are provided in
- Profiles for Australian Schools
- Mathematics key learning area in CSF.

The continua differ in
- the span of maths knowledge they cover.
- the extent to which they are based on 'expert opinion' vs researched.
- the size of each step.
- whether each provides a single description of each maths task.

Analysing mathematics performance. To complete any maths task, pupils need to

- think about the ideas in particular ways; to visualise and say them, model them link them with what is known, question them. These are the task processing strategies.
- manage the maths activity; they need to
  - plan how to manipulate the data, select the aspects of their knowledge they will use
  - decide when to paraphrase, visualise or represent concretely
  - decide when to pause and consolidate,
  - check that they are on the right track or take remedial action.

To complete any task, they use these strategies in the following order: they

- inform themselves of the task, that is, read the data defining the task. They must
  - comprehend the meaning of each element,
  - integrate or combine the meanings in the intended ways,
  - discriminate between relevant and irrelevant data,
- decide what the completed task will be like
- link the task to types of tasks learnt previously; they categorize it as an instance of types already learnt,
- recall and apply appropriate procedures to the data given,
- recall particular number facts,
- monitor the effectiveness of their efforts, and if these are judged to have been unsuccessful, to re-work the task.

The purpose is to identify the learning strategies students use while doing tasks. Ask them to work through tasks aloud or to report how they went about doing them ('reflective mathematics assessment). Note whether they

- use various strategies independently,
- know when to use each strategy,
- can apply the strategy to a range of instances or to simpler examples.

	Task 1	Task 2	Task 3
task orienting stage			
• say each symbol or element and what it means			
• describe the task, perhaps drawing or making it concretely			
• say what the outcome will be like			
• say how the task is similar to types of tasks learnt and say how they decided.			
• say what they will do first, second, etc.			
'implementing stage			
• apply the steps in an integrated, systematic way			
• monitor how they are doing the task, whether they are getting closer to a solution			
• recall relevant number facts; note how automatically this is done.			
review, consolidating stage			
• decide whether the outcome is reasonable, possibly correct			
• review what they have done in terms of its adequacy and if necessary take remedial action			

Assess what learners know about how to do maths tasks, what they believe to be 'good' or useful actions to use; use questionnaires and / or interviews.

Identifying the conditions under which the student corrects errors made. Note the cueing conditions under which the student corrects errors, for example completing a counting sequence similar to items 7, 12 and 13 on the Numeration subtest of the KeyMath.

If pupils initially answer incorrectly, cue them to think about the information with an increasing amount of 'scaffolding' and note when they can correct the error made. You can

condition under which error is corrected	implication for teaching
say the task, perhaps as "*mm, forty six, forty seven, mm, forty nine*	the student may not have said spontaneously the task to themselves
say the sequence for them and note whether hearing it said helps	they had difficulty translating the symbols into oral language form that could be analysed
describe the nature of the task	they could tell themselves the purpose of the task,
repeat the task instructions in alternative language forms	language comprehension accounted for the error,
What does this task remind you of?	they can link the task with what they know
break the task into component tasks and see whether they can complete each part in turn	they can break the task into "digestible chunks" and encode the information in short-term working memory
draw attention to the "maths actions" in the task	they can be cued to use relevant maths actions
when it is re-drawn vertically.	some perceptual arrangements are understood better
provide additional prerequisite arithmetic information	
mark the numbers on a number line and cover the deleted numbers	

Apply this type of analysis to maths tasks:

Condition under which the pupils can self correct	What this means about the nature of their difficulty and conditions for teaching
What does the task say ? Do errors in comprehending elements (symbols, words) explain learning ? If the pupils self correct after they • read each concept name or name each symbol; • say in their own words what each symbol means • see / select concrete or pictorial instances that may match the symbols or elements • hear each symbol verbalised • see components in compound symbols probed	the pupils may not have • said each symbol / concept in familiar oral language when attempting it • used their prior knowledge • comprehend term in the required way • read each symbol effectively, didn't convert it to an oral language form • understood the meaning of each position or difficulty discriminating between positions.
Relating the elements in intended way If the pupils self correct after they • say the statement • hear the statement said for them • picture the statement, represent it concretely • see a pictorial or concrete representation of it or hear real-life instances of it • describe it in their own words • hear or see an alternative written or oral word description	the pupils may not have • said the statement spontaneously • recoded the task to orally and used oral reasoning • spontaneously linked ideas meaningfully • understood the task description • linked the task to their general knowledge or didn't use their general reasoning • used the relationship between the symbols, the particular grammatical forms
Building an overall picture of the task If the pupils self correct after they • hear part of the task repeated, learn part of the ideas when you break the task into smaller meaningful "bits" • are told how to picture it, or when it in real-life contexts • see it expressed in small, familiar elements but not in more complex elements.	the pupils may not have • retained all of the information in short-term working memory • constructed a visual model of the data • met the reasoning skills demands
What type of task is it? If pupils categorise after they • are told to do so • when the task is given in a different format • when shown extra instances / non-instances • after hearing the task read • when aspects of the task are discussed • see task translated into concrete/ picture form	the pupils may not have • spontaneously self- instructed to classify • linked the task with existing knowledge • used all of the visual information • selected key information, used what they know • understood the abstract symbolic description.
Recall, apply appropriate procedure correctly after they • are told what type of problem it is • indicate what they would do first, second • have been told what to do • only after considerable prompting • detect errors made earlier or when errors are indicated • see it demonstrated for them	the pupils may have difficulty • recalling the appropriate procedures • both remembering where to start and actually doing each step • recalling the steps • steps not well integrated in pupil's memory

Recalling number facts when • the fact is related to other facts? • they do the procedure that produces the facts.	the pupils may have difficulty • taking apart the set of related facts • forming or recalling the complete set of facts

On task attentional and attitudinal behaviours towards maths. Learners' attitudes are displayed in how they behave or react to maths situations, for example

- whether they spontaneously approach maths tasks and problems.
- the kinds of maths tasks in which they are prepared to engage
- the level of enjoyment or satisfaction they display during or after maths, their curiosity.
- their on-task attention behaviours while doing maths
- whether they believe they are in control of their maths or display a level of helplessness
- what they think maths is like, its value and uses,
- their beliefs about how maths is learnt, and
- their self-concept or self-perception as learners.

Observing attitudinal behaviours in the classroom Note whether learners:

- spontaneously approach maths activities and choose vs avoid maths wherever possible ?
- show enjoyment or satisfaction versus a dislike for maths, show negative mood changes
- display curiosity and excitement versus anxiety and frustration during maths ?
- are interested / keen versus unwilling to show the outcomes of maths ?
- request the opportunity to read ?
- maintain interest versus easily distracted while maths ?
- show helplessness, increased dependence during maths, request excessive assistance
- seem less able to control and manage cognitive performance
    - show an ongoing dislike of and frustration towards maths

Learners attitudes to maths The types of information you might examine include

- what they believe about maths, its values, what maths is like, its value and functions.
- how they feel about maths.
- their self-concept as learners, whether they are likely to be successful as learners.
- what they believe about how to do math.
- their interest in maths.

Explaining maths performance ; how well can learner

- use symbolic codes ?
- recall the names of symbols automatically ?
- recall verbal versus nonverbal information
- store symbolism in STM.
- retain sufficient information in auditory short term memory.
- retain spatial information in short term memory
- operate analytically vs synthetically
- comprehend part-whole concepts both for sequential properties and equivalence properties
- sequence quantities, recognise a sequence
- dimensional thinking, for example, continue a dimension
- process spatial-numerical information, for example, sets of numbers or equations
- keep track of two categories or attributes at once, refer to two categories
- represent and perform mental actions of adding and taking away in different contexts

Referral data

Information from the learner's teacher and school

- past approaches and present approach to mathematics instruction
- whether the learner has the necessary prerequisites for learning to read, such as
    - visual capacities.
    - oral language and communication strategies,
    - cognitive strategies, for example, whether the learner remembers information
    - appropriate task-organisational strategies
    - previous experiences of the world and of mathematics.

- attitudes to mathematics; positive attitude to mathematics and to self as a mathematics student.
- the learner's general learning in the classroom, eg learning difficulties in other areas.

Information from learners. This includes

- how they see maths; what learners do when they learn maths, how they feel about making mistakes, to re-read, to guess at words they don't recognise immediately.
- how they see maths as a discipline, its values, how it can be enjoyable, interesting, useful.
- their emotional response to mathematics; whether they feel frustration, anxiety
- their self-concept as mathematics students.

Information from the child's parents.

- view of maths held by the learner, for example, whether learners
  - are interested in maths,
  - need to be forced to do maths tasks more than other homework tasks
  - give up on maths tasks more quickly than other tasks, or whether they spend inordinate periods of time trying to do maths,
  - achieved the appropriate developmental milestones in language and in other areas,
  - indicate an ongoing dislike of, and frustration towards, maths.
- view of maths held by parents, for example
  - Has the child been encouraged in the past to do maths ?
  - Have the learner's siblings shown an interest in maths ? Have there been other maths models in the child's home ?
  - Did the learner's parents find maths hard to learn.
  - Has parent assistance in the past clashed with approaches being used at school ?
- when did the learner first began to display maths problems and whether this emergence was associated with another event, for example, a physical injury or illness, visual problems ?
- the learner's developmental history. Parents can provide information about
  - whether learners achieved appropriate developmental milestones
  - whether learners displayed intermittent hearing loss
  - how easily learners communicated with peers in the preschool context, made sense.
  - when learners started and stopped the use of egocentric speech
  - the types of play in which learners engaged.

General referral information;

- when the teacher or parent first became aware of the child's maths problem,
- earlier steps taken to help the child,
- learning difficulties in other areas,
- involvement of other professionals in the child's learning history such as
  - sensory impairment ; for example audiologist, ophthalmologists
  - psychologists; information re developmental delay, overall general ability, social interaction,
  - medical; eg neurologist, psychiatrist, illnesses such as asthma, epilepsy, allergies, effect of medication on learning, effect of earlier traumas and injuries on learning,
  - motor development; implications of motor disabilities on maths.

Mathematics Evaluation Procedures K-2 North Sydney Region Infant Mistresses Council, 1979
Diagnostic Mathematics Tasks : Survey Tests for Junior Grades Burwood State College, 1980.
Booker, G (1995) Booker Profiles in Mathematics: Numeration and Computation. Melbourne : ACER .