Assisting students who have numeracy difficulties

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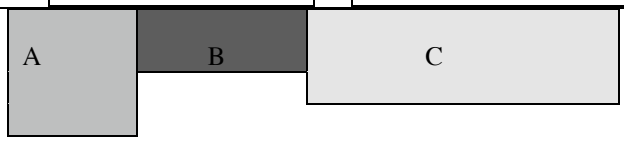
A recommended set of generic teaching procedures

Following is a set of generic teaching strategies intended to assist maths underachievers to learn maths more easily. The strategies are grouped under key 'learning functions' or interactions.

Challenge students to learn

Introduce new ideas as real-life problems to be solved, for example,	*I have two and a quarter pizzas. How many quarters.altogether ifI cut up the whole pizzas. What can you do to solve the problem ?*
Ask challenge questions.	Students decide questions the teaching might answer and what they think the answers are. *We are going to study subtraction. What questions might we ask about it ?* As they learn, they check their guesses against the teaching. The questions can guide their learning and as it progresses, modify the question sequence.
Present ideas that don't fit or seem to clash with what they know;	• *In running trials Peter ran .28 km in 2 minutes and Ann ran .3 km. Who ran faster ? By how much ? How will you decide ?* • *When you multiply, do you always end up with a larger number ? Multiply 8 by one half . Does this fit with what you thought ?* • *How are these different ?* *You know how to do this task* $\frac{1}{3} + \frac{2}{3}$ *How would you do this one ?* $\frac{1}{2} + \frac{2}{3}$
Have students guess and check; predict and compare their prediction with the actual outcome.	 *Which paddock covers the most space ? Which one has the longest fence around it*
Use novelty; make the novelty of new ideas stand out.	Present an idea in an unfamiliar context, for example, in a picture, a scenario from history or an animal can attract initial attention
Identify usefulness	Help them see how a particular procedure can achieve a goal or meet a need for them, overcome an obstacle, deal with a difficulty they are experiencing
Use open-ended tasks in which the students frame up questions and an action plan for completing .	*Write a brochure for gardeners explaining how to understand the difference between units of length, area , volume and mass*
use fantasy and imagination to lead students to possible questions and puzzles.	*Prepare a travel brochure for interstellar travellers telling them about distances, speeds, times in light years, etc.*

Have students invent similar problems for themselves, peers, share and to discuss their solutions, modify their attempts.

Every maths idea can be presented as a problem to be solved. For new ideas students can

- begin by solving problems,
- share and to discuss their solutions, modify their attempts
- invent similar problems for themselves, peers.

[1] This content is taken from Munro, J. (1995). SUCCESS in learning mathematics : A learning strategies approach. Hawthorn, VIC : EdAssist.

Introducing ideas in these ways motivates, fosters and validates curiosity and interest.

Students visualise the learning outcomes

Students say what they will know / be able to do having learnt the ideas.

Stimulate what students already know

- Help students activate what they know about a topic:

Experiential knowledge	• *What do you see in your mind/ think of when you hear percentages ?..?* • *When did you learn about how to find area ?* • *A feel for the ideas* • *Collect pictures of what you know about decimals.* • *Which of these pictures show a quarter ?* • *What can I draw / act out about the topic "time' ?* • *What is it usually like ?* • *Put the existing knowledge into a narrative, tell a story about metres.*
Abstract verbal knowledge	• *What do they mean ?* Check students' word-meaning readiness. • *Suggest synonyms and antonyms for counting numbers, add.* • *Brainstorm the topic . Draw a network map for multiplying.* • Students interview each other about the topic "division" • Ask students questions about the topic • *You write the article .* • *What can I say in 1 minute ?*
Action knowledge	• *What am I doing ?* Say the actions in words • Brainstorm the topic for actions • *What would you do when you hear ?* • *How would you do it ?* • *How would you say (* actions the students see done *) ?*

- synthesise what they know about the various aspects

	Idea 1	Idea 2	Idea 3	Idea 4
experiences				
words				
actions				

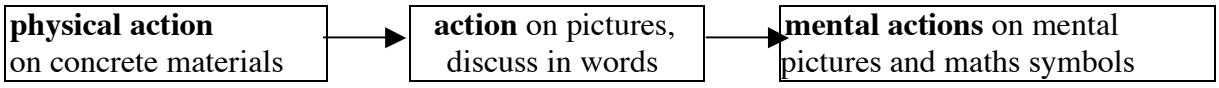
• Revise the factual knowledge they need to use automatically	A week or two before you begin to teach a topic have students revise the existing knowledge they will need to use automatically when they are learning the new ideas. You can do this using • quizzes, • maths card games, • sets of mental arithmetic tasks recorded on tape • class quizzes, partner quizzes.
• Help students see that they can learn the new ideas	Make sure students • see that what they know is useful and relevant to what they will be learning . • see that what they have been able to learn related ideas in the past • believe they can learn the new ideas and that we believe they can learn them as well. • know that it is OK to questions maths ideas, take them apart, try things out, see how ideas fit
• Help students get their knowledge ready to change	They can do this in several ways: by • saying the questions that their knowledge currently answers; • drawing diagrams that show how they think ideas are linked up • putting experiences into networks.

Change what students know

- ***Introduce new ideas in 'learner-friendly' contexts***. Maths teaching often introduces new maths ideas in unfamiliar symbolic formats. The new idea needs to be learnt in 'learner-friendly' formats first in real-world everyday events. They can learn to re-code the ideas to more abstract verbal and symbolic formats later. Learner friendly formats use real-life concrete materials that can be handled, pictures of the ideas and familiar symbols.

  Concrete and pictorial models of an idea bridge from students' knowledge of the real world to the more formal mathematics abstract representation.

- ***Allow students to build ideas in each code***
  - Help students to learn <u>how to act</u> out maths ideas, then act on pictures and then on mental pictures and talk about the ideas.



  - Cue students to think about the idea in different ways

Learn new idea in particular cultural, social or historical contexts	***Link new ideas in scientific-mathematical ways***	***Link emotions/ feeling with new ideas***
When were %s first used in history ? What cultures would be more likely to use percentages ? How will using calculators affect how we use percentages ? Will people in the future use percentages more or less ?	Can any fraction be converted to a per cent ? Apply % procedures Writing your own procedures for working out percent problems ? Prerequisite mathematics knowledge for dealing with numerical % problems ? Classify per cent problems in different ways Deciding which procedure to use ?	How would you feel if you were a number being per cented ? (you've been cut into 100 equal pieces) What feelings help people to solve % tasks? (curiosity, etc)
Draw attention to the cultural, historical aspects of ideas	learn ideas in symbols, abstract, ideas, to think about them in a general way	the feelings attitudes linked with ideas

Doing percentages

Link new ideas in words, in sentences, in more abstract ways	***Link new ideas in particular contexts and in images***	***Learn the actions that go with the new ideas***
Brain-storm ideas --> concept map--> network map. Discuss "What type of number is a % ? Is 20 % more like 20, 1/5 or .2 ? Show on a network map numbers whole part of whole fractions percents Think aloud working through per cent tasks. What are useful things to say ? Teach a friend how to solve % problems.? Other words to say 'per cent'? Where does the word come from ? Make up 6 difficult % tasks Debate : "Why bother to use % now ? They are unnecessary." Paraphrase % tasks	Collect, picture, draw, situations in which people use per cents in real-life 9% of population is unemployed rate at which interest is paid on a loan 50 % of the group means 1/2 of the group 25 % of the space means divide by 4 What do all instances have in common Invent icons of per centing such as the 100 piece bread slicer	What do you do to 'find a per cent' ? What is the per centing action ? Groups of students acts out something being 'per cented' Is there an opposite / reverse action to per centing ? If 200 ---> 2, what action undoes this, that is, takes you from 2 ----> 200 ? Make up models to show per centing. What are other actions like per centing ? What is special about per cent action ?
think about the ideas in words, paraphrase or summarise them, work on links between verbal concepts	Remind students to think about ideas in real-life contexts, visualise them	use actions to represent ideas, to imagine the ideas changing

- Teach students to switch ways of thinking or recode about ideas
- Students show what they know first in familiar ways (in words, pictures and familiar symbols).
- Account for global strategies in the teaching. Learners think about ideas in two main ways:

Serial - analytic strategies	Synthetic-global strategies
Work on bits of information	Look for overall patterns, scan,
Learn step by step, delay giving answer	Leap in and answer quickly, guess impulsively
Focus on detail and specific facts	Focus on overall idea, miss or ignore detail
Think in one direction provided by teacher	Think by moving in several directions at once
Take things apart, work on the parts	Think in wholes; don't take things apart
Follow other people's directions well	Prefer to direct, manage their own learning flexible in their thinking, unanswered questions.
Prefer less flexible convergent learning	Prefer flexible, open-ended learning contexts
Learn other's explanations, procedures	Prefer to work out own explanations
Analyse, sequence ideas in learnt ways	Arrange, sequence ideas less predictably.
Reflect about an idea often for a long time	More likely to 'guesstimate'

Learners use these two types of strategies through the self talk they use to guide their learning.

Teaching often favors serial - analytic strategies : these allows teachers rather than students to control or manage the rate at which students learn. Children who use serial analytic strategies find it easier to learn in teacher centred groups and are often easier to teach in groups.

Teachers need to take account of and foster both types of strategy in teaching:

Teach the broad, overall idea first • include open-ended tasks • let then see it is useful to take risks • encourage intuitive thinking	• What does this remind you of ? • What do you think about ? Why might …. Have happened ? • Use words such as *gut-feeling, 'just know', intelligent guess, possibility* • What is your opinion ?
Teach the specific details and analyse ideas	• What are possible ways of checking out your hunches? • How could you plan to research these possibilities to see if they do work ? • What causes /are the consequences of …? • What can you do /tell yourself to do first, second …? • What are the things that are most important / work best for ….?
'get above' the detail to see the 'big picture'	• What are the key parts of the ideas ? How do they fit together ? • Put ideas together • How would you plan for a similar topic / problem in the future ?
Examine the new idea from different perspectives to find out more	• link ideas in new ways, • look at ideas in novel perspectives, think flexibly • in unusual creative ways and reach personal solutions
Evaluate when each type of thinking is more useful	

Useful teaching strategies for these students include

- asking students to suggest their own solutions to maths tasks, allowing them to look for the overall impression first before teaching them the conventional idea, allowing them to put ideas together in unusual creative ways and reach solutions quickly.
- allowing students to learn some ideas first in their way and to manage their own learning in some situations
- teaching them strategies for attending to specific details and analysing ideas and for planning their way through maths tasks , while valuing their more global knowledge.
- having them work on putting their personal procedures into words (some may have difficulty keeping track of the thinking strategies they use). These can later be compared with the conventional procedures to be learnt
- learning to use the conventional ways of talking about their knowledge. Some may find it hard to organize their ideas in acceptable ways or give acceptable explanations.

- Teach the conventional symbolism for reading and writing maths.

- Develop maths learning as a co-operative activity in which students share knowledge, work together to solve problems, act on each other's knowledge and contribute to the group's understanding and appreciation of maths. Many children see maths as an individual activity, in which they learn alone. They need to learn maths in group activities in which ideas are exchanged. Use group learning in which students

    - co-operate to solve problems, build word definitions,
    - write problems and mock tests for other groups of students,
    - share their maths ideas, discuss ideas with peers, work in groups to decide what questions might be useful to ask about a topic to be learnt,
    - take turns to be the teacher in explaining or justifying an idea, writing problems, suggesting how maths ideas occur in everyday life or in hobbies,
    - discuss how they might solve or solved a problem, share with others strategies they used,
    - discuss what were the main ideas in a topic, the best ways of studying it,
    - use maths games and related activities, develop their own games that give them the opportunity to apply their knowledge
    - engage in reciprocal teaching procedures for learning new maths ideas .

There are several related attitudes that we can model in our teaching. We can show them that
(1) at the beginning of a task, we don't have all of the answers, but that, by discussing, trying out ideas, deciding what questions to ask, together can solve the task.
(2) we are keen and motivated to change our knowledge of mathematics and that the working together is not only for our students' benefit but is also helping us.

Abstract the ideas learnt.

- Help students organize their maths knowledge into categories
- Teach students to link the ideas with what they know.
- Teach episodic, network, procedural and affective aspects of an idea in an integrated approach
- Teach students to analyse and generalise the ideas ?
- Encourage students to question maths ideas : Ask them different types of questions and have them ask questions of the ideas.

- ***Help students organize their maths knowledge into categories*** Many students cannot effectively use what they have learnt. They have difficulty

    - seeing how a math task is like types of tasks they have learnt previously,
    - deciding which procedure is most appropriate,
    - solving word problems; they mis-interpret them and use inappropriate operations.

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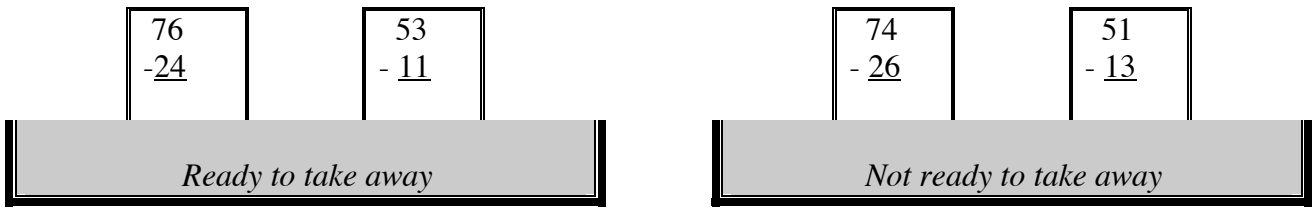
They need to learn how to organize their knowledge into categories. Categorisation or classification activities need to be included within the teaching. One useful way of categorizing mathematics ideas is to use the ready / not ready categories;

Examples



This is not the only categorisation scheme useful in maths. Students can learn to classify

- word problems into instances of addition, subtraction, multiplication and division,
- number facts into categories such as

addition facts to10	ties	adding 9, 8,7

- perimeter and area tasks ;

distance around the edge	Area
• perimeter of a rectangle • circumference of a circle	• finding the space inside • area • surface area

To help students to learn to categorize they can

- categorise instances of each type and gradually categorize 'in their heads'.
- decide how instances of each type are alike and differ , discuss shared properties,
- describe what they will look for when classifying instances of each type
- produce their own examples of each type.
- make patterns stand out, make the key aspect as salient as possible (using colour, a shared name or action).

- ***Teach students to link the ideas with what they know***. Help them slot new idea into an existing network by linking them with more general / specific ideas. Teach them to draw a picture of how their networks are changing. Use this to help them see
  - what they know and how they will change it when next learning related ideas,
  - how they are making progress, how their knowledge is changing,
  - how the set of ideas in an area fits together, how ideas link up
  - how to use optimally conceptual / procedural / episodic knowledge
  - how to prepare for tests
  - where their knowledge might go in the future.
  - how they can link experiences with the ideas in networks

- ***Teach episodic, network, procedural and affective aspects of an idea in an integrated approach*** Analyse topic areas that you teach and target all aspects:

  - episodic: help students to see how maths relates to the real world, to build episodes. *In which contexts does it arise ? What ideas occur with it ? When is it used ? What images / icons can I use to remind me of it ?*
  - network : *What are more general / specific ideas is it linked with ?* Students can extract the main ideas from specific problems and describe them as tentative rules.
  - procedural : *What actions characterise the set of ideas ?* Convert new ideas to procedures and action sequences.

You can diagnose and remediate difficulties in student learning; when students have
- networks without procedures they can explain why but not show how
- procedures without networks they can show how but can't explain why
- meaning network without episodes - they know 'why' but can't use or transfer ideas
- procedures without episodes - they know how to but not when to.

- ***Teach students to analyse and generalise the ideas*** The teaching can encourage students to
  - generalise across episodes, compare them, note what they have in common, how they differ, identify the essential aspects of the ideas.
  - summarise the ideas: *Say what you know now Write a summary / a table* . They record key ideas progressively in notes, key words, underline key ideas.
  - develop a general prediction or expectation about the ideas, a procedure or formula.
  - suggest the types of contexts to which an idea or procedure can be applied.
  - synthesise the ideas in other ways.

  - look at ideas from various angles, for example,
    - the positive and negative aspects of the ideas; What are good / bad things about using calculators a lot to do maths ?
    - how the ideas might be used in the future; What new things might calculators be able to do in the future that they can't do now ?

- ***Encourage students to question maths ideas are learning by asking them different types of questions and having them ask questions of the ideas.*** Use questions to direct attention to the ideas at hand and to attract interest. Questions can invite students to look at the ideas in different ways and extend their understanding. Types of activities include

(1) students convert the ideas learnt to question forms and suggest the questions the ideas answer, for example,
- when shown a worked task, they suggest the question asked
- what questions could you ask about adding fractions

(2) questions that ask students to infer; to reason about ideas, look for patterns, for example, *Why did I do ...?* for example, "*Why did I rename 43 as 30 add 15 ?*"

(3) questions that ask students to generalise or extrapolate for example, *Do you always* (get a smaller number when you divide)?*What would happen if* (the denominator gets smaller). ?" Rather than a one-off answer, students can modify their answers through questions that direct them to analyse parts of their answers, for example, "*How does .. fit in* ?"

(4) open-ended, non-directive questions that ask students to explore alternatives. "*What might we do here ?*" "*What do you think will happen ?* " You may follow up these questions with "*Why do you think that ?* " "*But what about .. ? Do you mean.. ?*"

(5) students inventing and answer questions about the ideas, making up mock quiz for peers.

(6) questions that link the ideas with existing knowledge, for example, "*What does this remind me of ? What did I do when I had similar learning tasks in the past ?*"

Help them to see how answers and questions are related.

Students examine what helped them to learn

Help students improve how they go about learning by reflecting on what they did to learn, the learning or thinking actions (or strategies) they used. Many maths underachievers don't know what to do to learn maths- they don't use spontaneously a range of learning strategies. Useful teaching procedures include the following

- ask students *What did you do to learn the new ideas ? Did making a picture help ? What new things have you learnt about how to learn ?* They discuss whether visualising or verbalising helped, how they planned how to tackle a task.
- have students 'think aloud' in various subject areas as they work through tasks and learn the language for talking about their thoughts.
- teachers can model, demonstrate the ways they learn.
- students keep track of the learning actions that work best for them, when to use each one.
- students share what they do as they work through tasks and trial each other's actions, see how and if these help, add to their set of 'ground rules for learning, for example
- suggest particular strategies in an integrated way and have students try them out, see if they help them and if they do, keep a record of them and remind themselves to use the strategies in the future.

Mathematics learning strategies include:

(1) problem solving strategies that students use to guide themselves through tasks / problems,
(2) symbol reading strategies that they use to comprehend number statements eg '5+ ❑ = 9',
(3) pattern analysis strategies, that children use to analyse patterns in mathematics,
(4) organisational strategies, that children use to categorise ideas and to discover how ideas may be similar or different, and
(5) short-term and long-term memory strategies.

Strategies for learning an algorithm. Students learn to tell themselves what to look for when needing to learn an algorithm or procedure. When the algorithm or procedure is presented in a set of symbols, they need to break the set of symbols into meaningful bits, extract the steps or actions and to synthesize these into the algorithm. They need to tell themselves to

- analyse the symbolic array,
- look for a certain number of steps *"I have to learn a procedure that involves .... steps. I need to look for each step and to put the steps in order.*
- *encode each in short-term working memory: "I will say each step to myself"* They can use a mnemonic such as a vertical set of hooks or shelves and visualise putting each step on one hook.
- combine the steps into an action sequence : "*Now I need to say all of the steps in order.*" It is often useful to do this with a worked instance present. "*The first step: First of all I..... The second step: Second I .....*"
- note when to use the procedure. "How is this type of problem like other problems that I have already learnt? How is it different ? How can I decide when to use it ?"

Applying a procedure, or working through a task or problem. Some students have difficulty getting started on maths tasks, deciding what to do, 'get lost' while applying a procedure, ignore key information, use inappropriate procedures. They need to learn to direct their attention to specific aspects of the task at a time in a systematic, organised way:

What does it tell me ?
Will reading it aloud help ?
Will drawing a picture help ?
What do I have to find ?
What type of problem is it ?
What does it remind me of ?
How is it like problems I have done ?
What procedures will I use ?
What does the first / second / third part say ?
Does the answer seem right ?
Have I answered the question asked ?

They can write the key questions on an index card and practise them with unfamiliar or difficult tasks and practise applying them 'in their heads' (or mentally) to problems; this will help them to

(1) speed up the time that it takes them to solve problems.
(2) see their way through problems,
(3) see possible 'danger areas, dead-ends' etc.' coming up.

Students see themselves making progress in learning

Many underachievers don't monitor their maths learning successes or see themselves making progress. This limits them from seeing that they can control and direct their own learning. We need to build in teaching procedures that will help them see themselves progressing and to recognise gains they have made. Useful teaching procedures include

- *What have you learnt ? Say, write, draw, demonstrate*. Students record this in a diary or log.Students record in a diary or journal the maths ideas that they have learnt, for example

  14/7 Today I learnt to do word problems about going on a picnic.

16/7 We worked on take-aways. Some of them were:

862	916	I have to remember to say each
-395	-253	column to see if it is ready to take away.
____	____	
____	____	

17/7 We did a table quiz. I got seven right.

They can also record situations in which they use their maths knowledge, for example,

10/7 I bought a new bike tube I measured the front wheel. I worked out the change from $20.

11/7 We worked out what time we would have to leave home by to get to the pictures on the train by 2.30. We had to use the train timetable.

- *What can you do now that you couldn't do earlier ?* They tell each other .
- They tick off on their journey where they are now.
- Use devices such as the following format to show the knowledge that they believe they know well and the ideas they are still allowing themselves to learn. New tasks are written on cards. Learners gradually move cards across as their proficiency improves.

New task	Not sure	More sure	Really sure	Do task perfectly
				2x+3= 9
		3(x-1) = 4		

or for the tables:

New table	Not sure	More sure	Really sure	Know table perfectly
				2 x table 3 x table
		9 x table		
7 x table				

Learners manage and control this system; they decide when a type of task is ready to be moved.

Many students do not rationally monitor their maths successes and continue to see themselves as unsuccessful. When learning a new idea, they can be told :

"*Yes, it is difficult for you now, but you know that you can learn it.*"

"*Last week you couldn't do ...[subtraction of fractions, for example], but you can do it now.*"
"*Twenty minutes ago you couldn't do .... but you can do part of it now.*"
"*You can't do this now but in ten minutes, you will know more about it.*"

They may need to have their past successes brought to their notice frequently.

Students respond emotionally to the ideas that they have learnt

Many maths underachievers never get the opportunity to respond emotionally to maths learning. Students will be more likely to be motivated to learn a set of ideas in the future if you have them link positive feelings or emotions with the ideas when they learn the ideas now. When their knowledge of the ideas is retrieved, you want it to tell them that

- the ideas are useful and / or interesting to learn
- they can successfully learn them
- learning the ideas will help them along their journey

Students decide whether they can achieve particular goals quickly and unconsciously. Their judgments influence how they learn. and are independent of the students' actual level of ability. We need to help students

- form positive beliefs about how well they can learn.
- attribute their success and failure functionally
- see themselves in control of their learning.

Having learnt a set of ideas, have students

- answer the questions "*How do you feel about .. ?*" "*Did you find this interesting/boring/ to learn ?*" These questions ask students to share their emotions while learning particular ideas. Many students are reluctant to talk about their feelings and need to be encouraged to describe how they feel about maths; the parts they like /don't like, find interesting or exciting, what they like doing, or don't like doing.

- see the usefulness of the ideas, comment on how interesting and / or useful they found the ideas. What would make them more interesting ?

- value themselves as learners, that it was their activity that led to them learning, draw their attention to what they 'had in their heads' and what they did, for example, when they had a difficult problem or challenge, they had a great deal in their heads that helped them to solve at least part of it.

- learn to give themselves positive feedback such as "*Praise yourself for a job well done*" before they begin to learn and to evaluate how well strategies worked for them; "*I tried hard, used the strategy and did well*".

- comment on how they are feeling, that it is acceptable to say "*This isn't bad; I feel I am making progress*", that they are doing things they couldn't have done earlier

- develop positive attitudes to learning; teachers can show a valuing of
  - curiosity and a preparedness to enquire in students
  - students can be given the opportunity to study and to reflect on how they learn.

- attribute success and failure functionally, that they see they did things to learn an idea and that statements such as *I didn't do well because I'm dumb / hopeless /could never learn this* or *I was lucky / got help / It was a fluke/ I had a good teacher* are countered by reality testing.

- reflect on various aspects of mathematics; they can comment on their mathematical experiences by maintaining for them a journal in which they record
  - recent mathematical experiences that they recall
  - their feelings about mathematics they are learning
  - questions that they might like to ask about the topics they are learning.

- see themselves as masters of numbers, that they can control the maths ideas and can make them work for them. To achieve this

  - encourage students to look for examples of maths ideas in everyday life, and to use maths procedures to solve problems that arise for example

    "I want to save up to buy a pair of skates that costs $38. I can earn $7.40 a week from my paper round. How many weeks will it take me, if I save all my wages?"

    "I want to get to the football about an hour before the game starts. It takes the tram 35 minutes to get from my stop to the stop near the ground. I live about 10 minutes from my stop. What is the latest time that I should leave home by?"

    "What are the changes of winning Tattslotto?"

  - apply maths to a theme or interest , for example, a hobby or a holiday or trip, see how math ideas can be used to solve problems in the area. Newspapers are a valuable context here. Maths can be used to solve problems, to model relationships, for example numbers used in the recent Voyager 2 trip, in various occupations.

  - games and related activities, particularly games that students make or modify.

  - the history of maths, famous mathematicians, their contributions, the conditions under which they worked,

  - encourage estimation in real-life situations, for example,
    "How many steps would you take to get to Fred's house ?",
    "How long would to take you to ride your bike to the oval ?
    "How much milk do you drink each week ?"
    Students estimate, check and discuss how estimates could have been closer.

- ***Take steps to reduce the likelihood of maths anxiety*** Maths anxiety is caused when students feel threatened. They may believe that they will -
  - lose self-esteem in the eyes of significant others (teachers, peers, parents),
  - be less valued, be seen as less acceptable, less intelligent,
  - lose access to future vocational opportunities, future study paths,
  - be less able to learn maths in the future.

Maths anxiety is learnt; it is

- not necessarily linked with low maths achievement or gender but it can debilitate performance,
- more likely when the learning assumptions made by teacher clashes with how student learns,
- can be overcome by helping students to see themselves as successful learners.

Maths Anxiety is shown in students'

- perception of maths as a discipline;
- perception of the maths teacher
- emotional response to maths learning
- perception of maths learning
- preference for maths
- self-perception as a maths learner

Why maths anxiety? Attitudes of students about maths and maths learning.

- Maths is a mystical set of ideas
- I am less acceptable if I can't learn maths.
- I need maths for my future.
- Maths means learning rules.
- You must avoid errors.

Practical ways of reducing maths anxiety :

First, we need to de-mystify maths and help students to see that:

- they can learn in a meaningful way; introduce algorithms as quantitative problems, allow students to express their understanding in familiar formats, talk about maths ideas .
- maths can be useful in solving personal problems, and
- ideas are organised and predictable, rather than random and disorganised.

Second, we need to change how students sees maths being learnt: that they:

- can look for patterns and relationships; comprehensible patterns underlie maths facts.,
- are free to make errors, and that these can be used for further learning,
- can learn how to learn maths effectively,
- can learn maths by solving problems.

Third, we need to help students to see themselves as a successful maths learners and to increase their self-concept and confidence as a maths learner. This involves -

changing how they sees maths and maths learning, being a master of numbers
helping them to learn how to realistically monitor their learning.

- **Students takes responsibility for their learning**

Underachieving students need to take control of their maths learning. Teachers can encourage this by having them

- encourage them to produce their own problems and problems for other students
- translate symbolic problems into concrete or real-life problems, and vice versa, e.g.
- offer parts of explanations and to suggest alternative ways of writing instructions.
- collect maths ideas from newspapers and the home, suggest ways in which maths ideas arise in the kitchen, or the garden, a maths notice- board in the classroom, or a maths day once a month, to which students are invited to contribute, can help.
- co-operate in negotiating their action plans, learning programs

Encode ideas in long term memory

Store information in memory
Retrieve information from memory by reconstructing it Allow gradual reconstruction
Storing information in memory

(a) "*I need to remember a new idea / procedure/ type of problem*". Students prepare themselves to remember, or 'cues themselves in' .

(b) "*I need to say what I am going to remember as briefly as I can*". Students say concisely what they are going to remember. If there are several ideas, they can compress them into 3 or 4 major steps or parts. They can
(1) write a summary card showing the main ideas and
(2) draw a concept map or flow chart showing how the ideas are related

(c) *What do the ideas remind me of? What are they like that I already know ? How are they different ?* Students link the idea with ideas already learnt. Drawing a concept map, categories charts will help, seeing familiar ideas and processes in the new idea, for example, "*Where have I done things like this before ?* " all help.

(d) "*What can I use to help me to remember this idea?*" Students use a distinctive drawing, gesture or a descriptive term to represent the idea.

(e) "*What is the use of the idea, what does it allow me to do?*"
(1) "*What does it do ? Why it is useful ?*"
(2) "*When will I use it in the future ? What type of problems it will help me solve ?*"

(f) "*I imagine myself remembering the idea*" Students imagine themselves remembering the idea at a future time.

Retrieving information from memory by reconstructing it Allow gradual reconstruction of the information;

(1) *begin with a recognition task; "We were working on a times table last night. Was it the four times, three times, or ten times table ?"*
(2) *check that they know what to do to produce the items*, This is a very important intermediate stage in getting to the automatic recall.
(3) *ask them to produce the complete set of related items,* in this case the whole table, in the least mentally demanding situation. In this case it would be writing them.
(4) *ask them to say the set of facts aloud, either by straight recall or by using the action* (in this case, counting on three). This is more mentally demanding than (3).
(5) *start to break up the complete set in the less demanding context.* Write down the set of facts, cross out every second one and have them answer the remaining items.
(6) *start to break up the complete set, in the more demanding context.* Have them say aloud every second item, perhaps taking turns to say successive items.
(7) *break up the complete set further, in the less demanding context*. Give them four or five items from the table.
(8) *break up the complete set further, in the more demanding context.* Say any item in the table, and have them say the next item,
(9) *recall individual facts, first in the less demanding context* Give them a set of mixed up items in writing.
(10) *recall individual facts in the more demanding context* Ask items out of order.

This reconstruction process usually take less than 10 minutes. You can repeat it in an abbreviated form, at the start of the next 2 or 3 sessions, with the students doing more of the work each time.

Automatize aspects of ideas learnt

The amount of thinking space that each idea takes up depends on the extent to which it is automatised. The more information we recall automatically, the more attention we can give to new ideas. Many maths underachievers do not automatise key aspects of their knowledge, for example, the tables.

Areas to automatize mathematical information

(1) addition and subtraction tasks to 20, tasks involving tens and hundreds
(2) multiplication table tasks and division tasks based on the tables,
(3) counting in increments of 2, 3, 4, ... 10,
(4) place value to 100 and then to 1,000,
(5) fraction tasks involving small numbers and the number of parts that make one whole and
(6) decimal tasks involving converting the simplest most common fractions and percentages to decimals, and vice versa.

Learning by rote' learnt an idea as an act of faith; they have no way of checking its truth.

To help students automatize their recall of mathematical ideas have them

(1) learn the ideas in an attention demanding way and do mental actions first ; they show they can learn the ideas.
(2) recall the ideas often, verbalise them and use them in a gradually wider range of contexts.
(3) recall the ideas more and more quickly, speed up using them and anticipate when they may need to use them.

Movement through this sequence is determined by the students.

Once you have identified the key aspects of a set of ideas you intend to have students automatise, you can design activities in which students

- speed up recall and deciding when to use the ideas, for example, in matching games
- practise repetitively parts at a time
- automatize links between ideas, for example, in rapid quizzes
- include automatizing activities in our regular teaching
  - regular revision of key ideas
  - rapid exposure to and processing of ideas
- emphasise semantic links between ideas so that one idea is more likely to activate related ideas
  - build links between ideas "If ... happens, it means ....."
  - draw network diagrams of related ideas, hierarchies, concept trees
  - students practise recalling links between ideas

Evaluate, assess learning outcomes

Some maths underachievers have difficulty showing what they know for assessment purposes. They need to learn the characteristics of the formal assessment formats in which they will show what they know; in tests, extended essays and projects. To do this most equitably, they need the

opportunity to align what they know about a maths topic with the assessment format. If the assessment tools

- are identifying small 'bits' of knowledge at a time as in short answer tasks, students need the opportunity to organise their knowledge into matching bits.
- are in verbal form, students whose knowledge is largely in nonverbal-imagery form need the opportunity to recode their knowledge into verbal form by talking about it.

Useful teaching procedures include having students

- discuss how they believe they will be expected to display the ideas in the future
- work in small groups to write assessment questions for peers, make up mock examinations.
- practise recalling the ideas.

Learn new idea in particular cultural, social or historical contexts	*Link new ideas in scientific- mathematical ways*	
Draw attention to the cultural, historical aspects of ideas	learn ideas in symbols, abstract, ideas, to think about them in a general way	the fe

Idea to be taught :

Link new ideas in words, in sentences, in more abstract ways	*Link new ideas in particular contexts and in images*	*Learn*
think about the ideas in words, paraphrase or summarise them, work on links between verbal concepts	Remind students to think about ideas in real-life contexts, visualise them	use ac chang

Learn new idea in particular cultural, social or historical contexts	*Link new ideas in scientific- mathematical ways*	*Link emotions/ fe*
When were %s first used in history ? What cultures would be more likely to use percentages ? How will using calculators affect how we use percentages ? Will people in the future use percentages more or less ?	Can any fraction be converted to a per cent ? Apply % procedures Writing your own procedures for working out percent problems ? Prerequisite mathematics knowledge for dealing with numerical % problems ? Classify per cent problems in different ways Deciding which procedure to use ?	How would you f being per cented ? equal pieces) What feelings hel (curiosity, etc)
Draw attention to the cultural, historical aspects of ideas	learn ideas in symbols, abstract, ideas, to think about them in a general way	the feelings attitu

Doing percentages

Link new ideas in words, in sentences, in more abstract ways	*Link new ideas in particular contexts and in images*	*Learn the actions that*
Brain-storm ideas --> concept map--> network map. Discuss "What type of number is a % ? Is 20 % more like 20, 1/5 or .2 ? Show on a network map numbers whole part of whole fractions percents Think aloud working through per cent tasks. What are useful things to say ? Teach a friend how to solve % problems. ? Other words to say 'per cent'? Where does the word come from ? Make up 6 difficult % tasks Debate : "Why bother to use % now ? They are unnecessary." Paraphrase % tasks.	Collect, picture, draw, situations in which people use per cents in real-life 9% of population is unemployed rate at which interest is paid on a loan 50 % of the group means 1/2 of the group 25 % of the space means divide by 4 What do all instances have in common Invent icons of per centing such as the 100 piece bread slicer	What do you do to 'fin per centing action ? Groups of students acts cented' Is there an opposite / r centing ? If 200 ---> 2, what action ur you from 2 ------> 20 show per centing. What are other actions What is special about p
think about the ideas in words, paraphrase or summarise them, work on links between verbal concepts	Remind students to think about ideas in real-life contexts, visualise them	use actions to represent ideas changing

Learn new idea in particular cultural, social or historical contexts	*Link new ideas in scientific- mathematical ways*	*Link emotio*
When were equations first used in history ? What sorts of equations might be used in different cultures ? How will using calculators affect how we use equations ? Would people in rural areas use different equations from urban people?	What is a general procedure for solving equations ? How do you decide what procedures to use ? How do you decide if a possible solution is correct ? What types of linear equations are there ? Can you see different ways of categorising them ? What are different ways of writing the solution ? How are equations different from other number statements ?	What feelin were an equ (you've beer you've been What feelin equations ?
Draw attention to the cultural, historical aspects of ideas	learn ideas in symbols, abstract, ideas, to think about them in a general way	the feelings ideas

SOLVING LINEAR EQUATIONS such as 4 + 2x = 7

Link new ideas in words, in sentences, in more abstract ways	*Link new ideas in particular contexts and in images*	*Learn the action ideas*
Brain-storm ideas -----> concept map -------> network map Paraphrase equation and the tasks; can you solve it by saying it? Imagine you are an equation ; describe where you came from ? Imagine you are 'x' being interviewed. What would you say ? Teach a friend how to solve equations. How do you explain it? Make up 6 difficult solving tasks? What are other words for 'solve'? Invent some word equations.	Imagine, picture, draw, collect situations in which people solve equations in real-life working out change sharing out lollies at a birthday party cost of petrol to finish a journey Draw a comic strip showing the steps in an equation being solved. Useful icons for solving equations ?	Make an action r What would you unknown ? Small groups of something being Is there an oppos solving ? What are the acti equations ?
think about the ideas in words, paraphrase or summarise them, work on links between verbal concepts	Remind students to think about ideas in real-life contexts, visualise them	use actions to rep the ideas changi

Learn new idea in particular cultural, social or historical contexts	*Link new ideas in scientific- mathematical ways*	*Link emoti*
When was place value first used ? Why did it arise ? What problems did it help solve ? What other numeration systems have been used ? Compare place value with writing Roman numerals/ What different types of place value have been used over history ? Is idea 'taboo' in some cultures? Will idea be used in future ? .How has the idea changed with other changes for example, calculators ?	Convert numbers to place value form. Distinguish between place value and other statements ? What is characteristic about place value statements ? Prerequisite mathematics knowledge for dealing with place value ? When do we use place value ideas ? Identify the assumptions, logic, on which the idea is based Link idea with related math-scientific ideas. Write idea in math-scientific language	What thing interested i What feelir place value How could interesting
Draw attention to the cultural, historical aspects of ideas	learn ideas in symbols, abstract, ideas, to think about them in a general way	the feelings

PLACE VALUE IDEAS BETWEEN100 & 10 000

Link new ideas in words, in sentences, in more abstract ways	*Link new ideas in particular contexts and in images*	*Learn the actior*
Develop aspects of place value in a story. Brain-storm place value ---> concept map --> network map. Discuss "What type of idea is it ? How is it like what I already know ?" Synonyms, antonyms. Slot place value into a network map Verbalize the symbolism. Write article about place value for school magazine Think aloud working through place value based tasks. What are useful things to say ? Teach peers to solve place value problems. Where does the word 'place value' come from ? Glossary Make up 6 difficult tasks / questions Debate aspects of place value.	Collect, picture, draw, instances of large numbers in real life and analyse them. Why is place value used here ? Reflect on, analyse specific instances ' Construct, build models for the ideas Discuss limitations of models Make videos of the place value Draw a comic strip of the sequence of decisions in interpreting place value, to show steps, etc. Draw diagram of place value. What do all instances share ? Invent icons of the idea Visualize, imagine new contexts in which place value can be used.	Do' the place val into tens and hur Organize quantit gathering instanc Look at the writt it into places Is there an oppos place value ideas Make up models Change a quanti other actions are special about the
think about the ideas in words, paraphrase or summarise them, work on links between verbal concepts	Remind students to think about ideas in real-life contexts, visualise them	use actions to rep ideas changing