# PROBLEM SOLVING

# GLIM

## Gifted Learners in Mathematics

Action Research Project

2005

**Aim of the Project**

To give talented and gifted students the opportunity to solve problems and then learn to effectively communicate their solutions

**Rationale:**

Gifted students often experience genuine difficulty in explaining solutions to their teachers and peers. Poor skills in this area lead to lowered success rates in formal assessment and related opportunities in the job market.

Helping students to develop skills in this area without ‘programming ‘ their thinking too much is the intention of this project.

**Process**

Groups of students in years 7 – 10 who were considered by their teachers to be talented/gifted were taken out of their normal class for one period per week over a series of three weeks and given a series of three graded problems to solve and then explain.

The importance of being able to communicate your higher-order thinking was discussed before the problems were started. Students were asked to write down what they were doing as they solved the problems.

Students were encouraged to work individually initially to find a solution to each problem.

They were then given the opportunity to try and formulate their thinking and explain what they had done to their peers.

## INTRODUCTION

Demographics of the group

- All boys
- Year 7 – 8 students
- Age 12-14 years

**How to identify GLIM students**

- By viewing their external testing results, but these results are not always accurate.
- These students tend to have their own methods and creative ways in which to solve problems.
- They don't always follow the traditional methods 'programming' and steps, They tend to come up with their own methods and approaches.
- These seems to be something different about them. They are sometimes 'quirky'.
- Tend to sometimes be 'social misfits'
- They sometimes tend to speak  a similar language and will often understand each other more easily. There may process information in a similar way.
- They tend to look for complexity where there is none.
- They often become easily frustrated when trying to explain concepts to others as they assume that everyone thinks the same way they do.

**TEACHING PROJECT**

Learning needs we intend to target

- Communication skills, getting students to be able to articulate their thoughts.

  Their way of thinking is so different

Planning for GLIM students

- Looking at he types of questions we present students with
- Giving them the opportunity to use their creativity

# *Case Studies*

Two students have been studied in depth because they have all been identified as gifted in most areas of mathematics and exceptional problem solvers. They are both able to solve problems that are beyond the reach of most of their peers, but experience difficulty in communicating how they reached their solutions.
In both cases a similar general procedure has been followed:
Helping them to understand why it is sometimes necessary to explain things to others- examinations, job interviews, working with other people etc.

- Asking them to explain their solution to a problem to a group of their peers.
- Going through with a staff member what they did and writing out a solution with them.
- Attempting to explain their solution to peers again.
- Giving them a series of similar problems to solve and asking them to solve them, take time to order their solution alone, and then explain what they did.

It is hoped that by going through such a process they will be assisted to order their thinking and explain things without the frustration that normally accompanies such attempts.

The problems attempted were solved as participants in the Year 7 and 8 maths enrichment groups which meet weekly and focus on problem solving strategies. Both boys have had a lot of exposure to similar problems and have successfully completed problems from the Euler series produced by the Australian Mathematics Trust in Canberra. The observations of these students are compiled from notes taken over a four week period.

A series of different problems were trialled:

**Problem One**:
Use the digits 5,7,9 and 2 once each to make a multiplication problem with the largest possible answer.

**Problem Two**:
Each letter in this problem stands for a different digit. Work out what each digit stands for. Try to find all possible solutions.

BACE
×4
CECDC

**Problem Three**:
Cori the camel lives at the edge of a desert that is 1000km across. Cori has 3000 bananas that she needs to take to the market across this desert. She can only carry 1000 bananas at a time and to keep going she needs to eat one banana at the end of every km travelled. What is the greatest number of bananas Cori can get to market?
All three problems were given to the students in the first session with no specific instruction as to how to solve them.
The first problem was solved quickly by both boys, and while their answers were not well articulated they could be understood. Both instantly recognized that to maximise the answer the larger digits needed the larger place value and they narrowed it down to a couple of possibilities which they checked to confirm.
The second problem was also solved correctly by both students very quickly, but not by systematically working out the value of each letter. Neither explicitly recognised that **C** could not be greater than three as a starting point like many of the others did, but they were both the first in their respective year levels to come up with a correct solution. Both were adamant that there were no other solutions but they could not clearly explain how they knew that.
Other students in the group were able to give much clearer explanations of how they solved the problem, but neither of the boys being studied were particularly interested in hearing the solutions of others because "they already knew the answer".
The most detailed observations are of the third problem and variations of it. Being able to change one or more variables and explore the effect on the solution, combined with the inherent uncertainty about whether the answer found was the best possible lent the problem to the sort of observations I was interested in.

**Student A**
Is a Year Seven student who struggles socially and has learning difficulties in some areas but is very keen and capable in mathematics. His organisational skills are poor and project work which requires a formal presentation is often left incomplete or of a very poor standard. He generally performs very well in tests but sometimes makes unexpected errors. His grades are not nearly as high as many of the other students in the group, but he consistently finds solutions to difficult problems before all of his peers.
With the 'camel' problem student A instantly recognised that the solution would involve making stockpiles of bananas in the desert. His written work was very hard to follow, but he did use a diagram initially to represent the situation. He firstly divided the journey into 200km sections, then tried 100km and 250km intervals.
Student A got a reasonable answer quite quickly, but was unable to explain it clearly and could not explain how he knew that his answer was the best possible. He was the only student to experiment with different distances in different legs, eg. 200km then 66km etc.

When he went through his solution with me individually he was able to explain more fully what he was thinking, but he had to be constantly challenged and questioned because he assumed that what he was saying was transparently obvious and further explanation was a waste of time.
After being encouraged to write out his 'best solution' in a more orderly fashion and then explain it to the group again he did a little better- I could understand him this time, but it still made little sense to his class mates. When listening to the explanations of his classmates he was dismissive of any methods different to his own and preferred to continue working on his own solution.
Some of the other students had solutions that involved the camel throwing bananas or dragging them along behind, but Student A was not interested in these more 'creative' interpretations.
Student A worked extensively on the problems in his own time between sessions and tells me he spent a lot of time in other classes where the work was 'boring' working on his own theories.
When the parameters of the problem were changed Student A adapted quickly, and could easily build on what he had learnt earlier. He was given a series of problems where the distance to the market, the number of bananas carried and the camel fuel economy were all varied. Student A also made up some where he determined the parameters and he made an attempt to develop an algorithm that would tell him how many bananas would make it to market under differing conditions.
He also spent many hours trying to write a computer program that would solve the problem for him. When he showed me what he was trying to do it actually helped me to get through to him the importance of being able to explain things in a clear and systematic way, because he understood that was what the computer needed. I could not follow much of his programming code, but he was more willing to try and explain things better after that.
So far, he has been unable to give a clear explanation of his thinking to his peers, but I am finding it easier to follow what he is saying. Being encouraged to explain his processes does seem to have helped him communicate his ideas but there is a long way to go. He still tends to be judgemental of the ideas of the others at times, but he has certainly gained more social acceptance, at least in this forum, over the last month.
Student A has a long way to go, but at least recognises the need to communicate more clearly and the benefits of ordering his thought processes as problems get harder.
I will continue to work with him along similar lines for the rest of this year and attempt to give him strategies to use when he is asked to explain something in class.

**Student B**
Student B is a Year Eight student who, like Student A is a bit of a social misfit. He does have a group of close friends who tolerate his 'quirkiness' and he has matured a lot socially since starting secondary school.
Student B is a gifted problem solver, but finds it inordinately difficult to explain even the simplest problem. When trying to explain how to solve a routine problem to another student his explanation is unclear as he seems unable to break it down into steps. He becomes easily exasperated, making incoherent noises and throwing his arms around in an unintentionally humorous fashion.
His organisational skills are poor and his writing close to illegible. This means that work is often not handed in on time and many of his grades are poor. He loves coming to enrichment mathematics classes where he is well accepted and admired for his

ability to get a quick answer. A lot of work has been done with him since he started at the school to help him get organised but there has been slow progress.
Student B solved the first camel problem quickly, but I did not even ask him to try and explain it to the class. I worked with him to tease out his thoughts and wrote his method for him. We then went through it to rehearse an explanation until he was confident. It was difficult for Student B to explain what he had done, but we were able to agree on an overall process that he had followed. When he explained it to the class he reverted to his usual ways about half way through, but it was a promising start. He was able to start a clear explanation, but was not able to cope with questions at all without getting exasperated.
At the end of the third session (near the end of the period) it was interesting that as Student B tried to explain his solution to one of the camel variants some of the Year Sevens were arriving for the change over. Student B was becoming frustrated that he could not make himself understood, and the only one in the room who seemed to have any idea what he was talking about was Student A who was standing at the door!
Student B is still resentful of having to explain himself at all, preferring to just solve the problem, but he has made significant progress. I have found that getting him to try and write down his process is too difficult, and the only way that we are making any progress is for me to talk him through the problem and write down the process for him. Even then he finds it very difficult to explain steps that he considers obvious.
In the last session he did get through an almost lucid explanation of one of the camel variants which was greeted by a spontaneous round of applause from the others. Hopefully the message is getting through.
Student B was able to solve these problems more quickly than anyone else, but even after extensive interviews it was unclear exactly how he did it. Without being able to express a clear method he was still able to solve subsequent related problems very quickly, so whatever was going on in his head was clearly effective.
I have not yet been able to convince Student B of the need to 'tidy up' his thinking so that others can understand what he has done. His focus is still on getting the answer, which he does very efficiently, rather than communicating the process but some progress is being made.

**Summary**

It seems ironic that the best problem solvers in each year level are among the least proficient when it comes to explaining their work. Student A and Student B share many characteristics, but both made significant progress over the month.
A lot more work needs to be done but the methodology seems valid, and the benefits for these students at school and in later life could be significant. The quandary is to help them to communicate in a logical way without losing the spark that they both have for solving problems intuitively.
Doing this work has clearly emphasised for us the difference between gifted learners and talented learners in mathematics. Some of the talented learners solved the problems efficiently and explained what they had done very clearly, step by step. Student A and Student B solved the problems much more quickly but found it very difficult to articulate what they had done, perhaps because they are more intuitive and do not go through steps in the same way.
To get them to explain their processes without 'programming' their thinking is the major challenge. The gifted are all individuals and the approach taken needs to be modified for each of them.