GIFTED LEARNERS IN MATHEMATICS

The activities utilized group work, peer teaching, internet searches, traditional teaching, exploratory learning (hands on activities), reinforcement, open ended problems and modelling.

A sample of activities included

Exploratory investigations	Deriving the Index Laws
	Expansion versus factorization
Open ended problems	Student designed worksheets
	Jigsaw Squares
Modelling	Which professions use equations?
Teacher directed activities	Traditional classes
Reinforcement	Traditional text exercises
Research investigations	History of algebra
	Famous mathematicians

Within all activities there was an emphasis on the terminology of algebra and the need to communicate ideas and answers clearly, logically and in a manner that was mathematically sound.

Impact of the GLIM Action Research

The adaptation and extension of the Year 8 Algebra curriculum into the Garden Beds format proved to be an innovative way to structure curriculum and prompted staff to look for alternative methods of presenting age old information using a variety of forums. It promoted discussion between staff, stretched our perspective and challenged us to pilot new teaching strategies. It was an excellent form of professional development for each of us.

The inclusion of a research tasks based on the History of Algebra and Life of a Famous Mathematician [1] was initially perceived by the students as unusual and somewhat strange. However it provided students with another perspective of mathematics and number including the historical and potential social impact rather than mathematics being merely a subject in a school timetable that students either did well or not so well at. The tasks also allowed students the opportunity to present their work in a range of non traditional methods (for a Maths class) including posters, PowerPoint presentations, oral presentations and even a dramatisation.

Curriculum implications

The Garden Bed curriculum format demands a multi disciplinary approach to learning and, as such, allows more scope for individual differences. It is proposed that at least one topic at both Year 7 and Year 8 be reformatted into Garden Beds curriculum so as to cater for a range of student abilities.

[1] Appendix 7 – History of Algebra Assignment, Famous Mathematician research guidelines.

One School's Approach to Garden Beds

Learn new ideas in particular cultural, social or historical contexts

- Research a mathematician from the list provided. Give an outline of his/her significant contribution to society. Present your findings in a creative manner

- "When are we gonna use this" cards
- "Equations and formulae around us"

Draw attention to the cultural, historical aspects of ideas

Link new ideas in words, in sentences, in more abstract ways

- Network map of known terminology
- Simplifying expressions – teach a friend; design a worksheet
- Expansion – teach a friend; design a worksheet
- Factorisation – teach a friend; design a worksheet
- Magic Squares – factorisation and expansion

- Difference between an equation and a formula
- Changing equations and formulae from
  symbol $\rightleftharpoons$ word
- Substitution – teach a friend; design a worksheet
- Solving equations – teach a friend; design a worksheet
- Dictionary of terms used in this unit

Think about the ideas in words, paraphrase or summarise them, work on links between verbal concepts

One School's Approach to Garden Beds

Link new ideas in scientific-mathematical ways

- Derive the 1st, 2nd and 3rd Index Laws based on the knowledge of multiplying and dividing algebraic terms
- Using knowledge of expansion to derive factorisation protocol
- Posters generalising rules for collection of like terms; multiplication and division of algebraic terms; Index Laws; expansion; factorisation

- Substitution – review of hidden signs; BODMAS
- Equations – backtracking; SAMDOB
- Modelling activities using generalised arithmetic and relationships

Learn ideas in symbols, abstract ideas, to think about them in a general way

Year 8 Algebra unit

Link new ideas in particular contexts and in images

- Comparison of rules for collection of like terms with addition/subtraction of physical quantities eg time, speed etc
- Expansion and factorisation using coloured squares
- Flowchart on "How to tackle a multistep expansion"

- Investigate "real life" substitutions
- Investigate "real life" equations
- Flowchart on "Solving a multistep equation"

Remind students to think about ideas in real-life contexts, visualise them

One School's Approach to Garden Beds

Link emotions/feelings with new ideas

- Create some equations or formulae that could be applied to the life of a student

The feelings and attitudes linked with ideas

Learn the actions that go with the new ideas

- Formal exercises on simplifying expressions to reinforce skills
- Formal exercises on Index Laws to reinforce skills
- Formal exercises on expansion to reinforce skills
- Formal exercises on factorisation to reinforce skills

- Formal exercises on substitution to reinforce skills
- Formal exercises on solving equations to reinforce skills

Use actions to represent ideas, to imagine the ideas changing

Skills and concepts summary

Assumed entry skills	May need to revisit……	New concepts to be developed
▪ Definitions of algebraic term, expression ▪ Addition and subtraction of algebraic terms ▪ Operations with directed nos., fractions, decimals ▪ Orders of operations	▪ Definition of equation, formula ▪ Multiplication and division of algebraic terms ▪ Formal presentation of substitution ▪ Solving simple equations and formal presentation	▪ Index Laws ▪ Expansion ▪ Factorisation ▪ Solving multi-step equations

Examples of Worksheets

Expansion

Step 1:

- Set out 2 Yellow and 3 Orange squares in a line
- Write an algebraic statement to represent this line

2y + 3h

Step 2:

- Make 4 copies of this line
- Write an algebraic statement to show the number of lines and what is in each line

4(2y +3h)

- How many Yellow? How many Orange?

8y +12h

Step 3:

- Write an algebraic statement to represent the total number of squares used

4(2y + 3h) = 8y +12h

Step 4:

- Now repeat the activity using your own numbers. Make sure you write the algebraic statements

Factorisation

Step 1:

- Count out 6 Yellow and 12 Orange squares
- Write an algebraic statement to represent this line

6y + 12h

Step 2:

- Place the squares into lines so that all lines are the same
- How many lines have been made?

3

- How many Yellow in each line? How many Orange?

2y + 4h

- Write an algebraic statement to show the number of lines and what is in each line

3(2y +4h)

Step 3:

- Write an algebraic statement to represent the total number of squares used

6y + 12h = 3(2y + 4h)

Step 4:

- Now repeat the activity using your own numbers. Make sure that you write the algebraic statements

Challenge **Can you derive the Index Laws?**

Like all branches of mathematics, algebra has its own set of rules or Laws. The Index Laws are one such set. There are 6 Index Laws in total. Today's challenge is to use existing algebra skills to derive the first 3 Index Laws.

Step 1: Solve the following problems. It is a good idea to show your working out initially but you should also aim to find the "short cut" or generalized pattern as you work.

$y^2 \times y^3$	$m^4 \times m^5$	$t^2 \times t^5$
$d \times d^5$	$s^2 \times s^2 \times s^3$	$h^2 \times h^4 \times h$
$\frac{y^4}{y^2}$	$\frac{k^5}{k^2}$	$\frac{z^4}{z}$
$\frac{j^5}{j^4}$	$\frac{p^4}{p^4}$	$\frac{r^2}{r^2}$

Step 2: Study the patterns between the question and the answers. Now generalize this pattern and use this to derive the Index Laws.

1st Index Law $y^a \times y^b =$ example: $d^3 \times d^4 =$

In words:

2nd Index Law $\frac{y^a}{y^b} =$ example: $\frac{p^7}{p^2} =$

In words:

3rd Index Law $y^0 =$ example: $g^0 =$

In words:

where a and b can be any power (index)

Step 3: The next challenge – can you extend the Index Laws to cover the following types of problems?

$5h^2 \times 3h^2$	$8y^3 \times 6y^2$	$4d^6 \times 2d^4$
$3r^2 \times 4r^3 \times r^5$	$4f^2 \times f^5 \times 3f$	$\frac{25d^7}{5d^3}$
$\frac{12g^8}{4g^5}$	$\frac{24h^6}{18h^3}$	$\frac{30j^6}{6j^6}$

1st Index Law $ny^a \times my^b =$ example: $3d^3 \times 3d^4 =$

In words:

2nd Index Law $\frac{ny^a}{my^b} =$ example: $\frac{10p^7}{5p^2} =$

In words:

3rd Index Law $ny^0 =$ example: $7g^0 =$

In words:

where a and b can be any power (index)
and m and n can be any co-efficient

The Super Challenge – Apply the Index Laws to solve the following problems. Compare your answers with those of another group – be prepared to justify your answers

$d^6 \times k^3 \times k^7 \times d^2$	$5f^4 \times 4d^2 \times 2f \times 4f^2 \times 2d^5$	$\frac{3r^5 \times 4r^5}{6r^3}$
$\frac{6p^3 \times 10p^7}{5p^2 \times 3p^2}$	$\frac{4y^7 \times h^6}{3h^4 \times 2y^3}$	$\frac{3g^6 \times 2s^4}{5f^3} \times \frac{5g^2 \times 4f^5}{s \times 10g^5}$

Jigsaw Squares

1. Designing the game

- Draw up a square 3 x 3 grid
- On another piece of paper write 4 expressions in both their expanded and factorized forms

Eg 3(2y + 6) and 6y + 18

- Place the pairs of expression within the grid, one on either side of the internal lines, so that they match up to each other along the internal lines.
- Now place an expression along each external edge so that they become the "red herrings" for the player – these will not be able to be matched up
- When all edges have an expression written on them, cut out the nine squares.
- Challenge a class mate to reassemble the jigsaw, matching the expressions along all of the lines

Sample

Note : be sure to place expressions on the vertical edges as well !!

6y + 18 3(2y + 6)	5(4y – 7) 6y + 18	20y – 35 5(4y – 7)
6y + 18 5(4y – 7)	3(2y + 6) 5(4y – 7)	20y – 35 3(2y + 6)
20y – 35 20y – 35	20y – 35 5(4y – 7)	6y + 18 5(4y – 7)

2. Objective of the game

- Arrange the nine cards in such a way that the expanded and factorized forms of the expressions are matched up along all internal lines
- The nine cards must form a square

3. Teachers notes

- Some students will use more complex expressions while others will use simpler expressions when designing their Jigsaw Squares
- The number of pairs of expressions that you ask students to use within their Jigsaw Squares will affect the complexity of reforming the square – the more pairs of expressions used, the simpler the square will be to work with
- This activity can be used in a range of topic areas within the curriculum

  Examples: $3y + 5y$ and $8y$ type problems
  -4×2 and -8 type problems
  $3y^2 \times y^5$ and $3y^7$ type problems

- Reconstruction of the Jigsaw Square requires more than a knowledge of expansion and factorization – it also requires students to utilize skills in transformation and problem solving and to persevere!